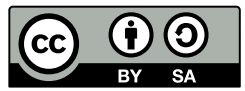
Paskaita bus įrašyta
This lecture will be recorded

Tranzistoriai ir loginiai elementai

Saulius Gražulis

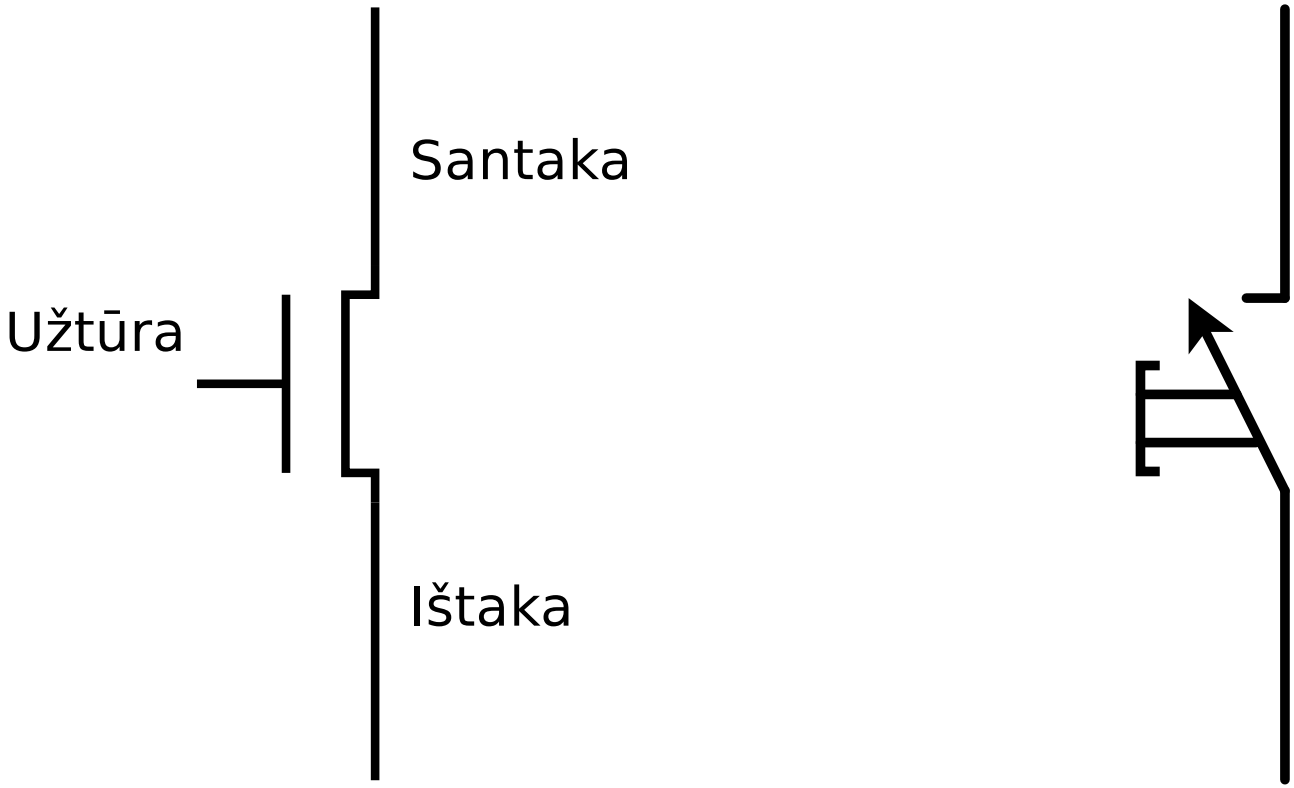
Vilnius, 2020

Vilniaus universitetas, Matematikos ir informatikos fakultetas
Informatikos institutas



Valdomas perjungiklis – tranzistorius

FET – Field-effect transistor
MOSFET – Metal–oxide–semiconductor field-effect transistor



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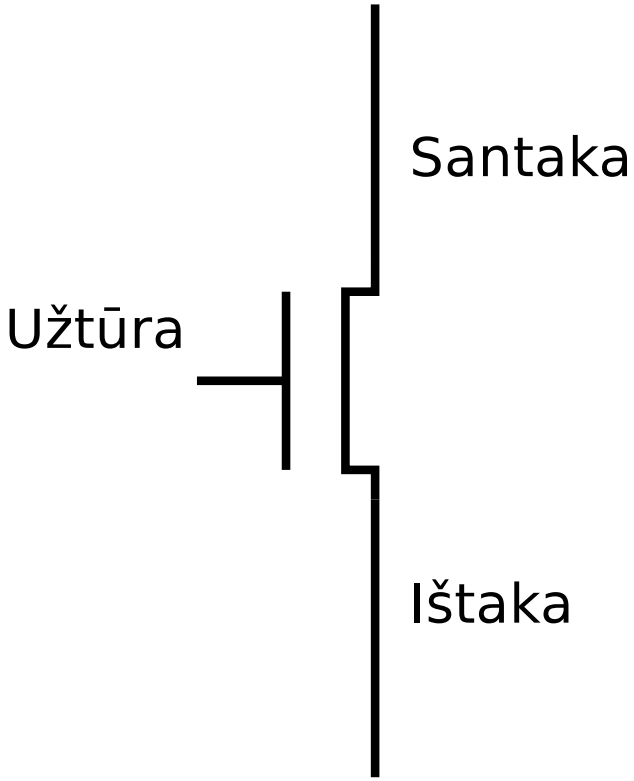

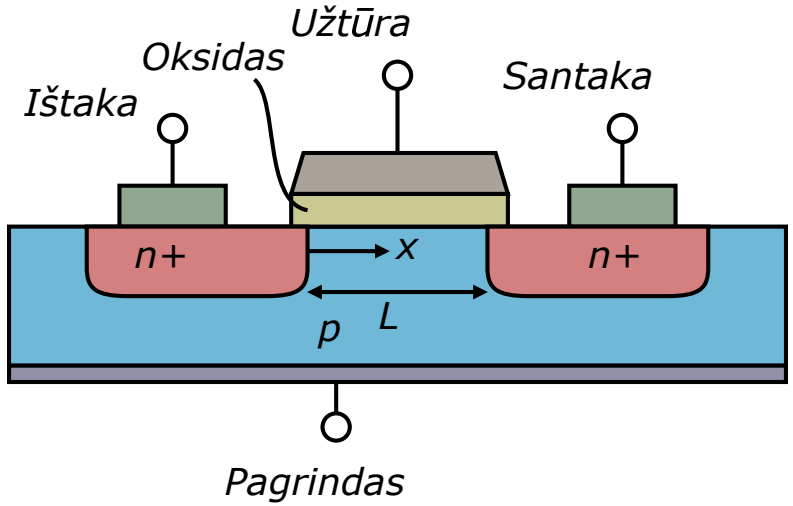

Cyril Buttay [CC BY-SA 3.0]

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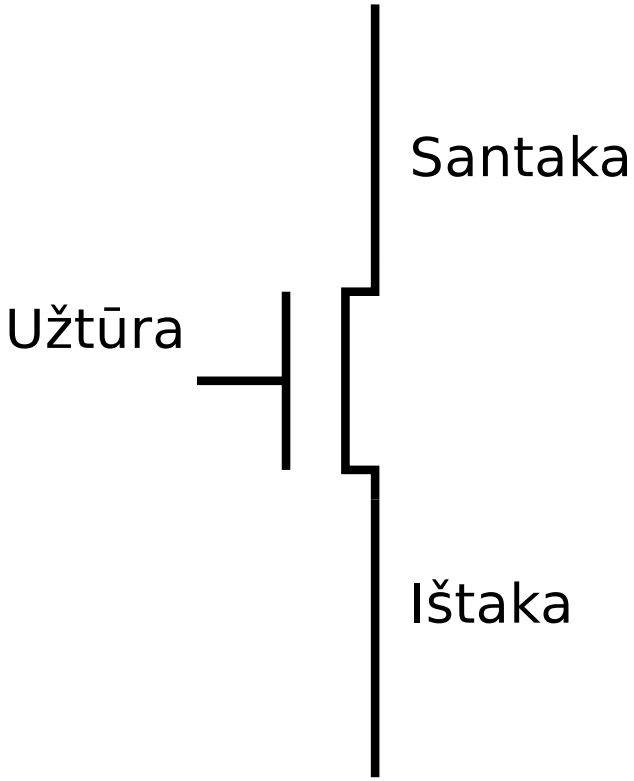
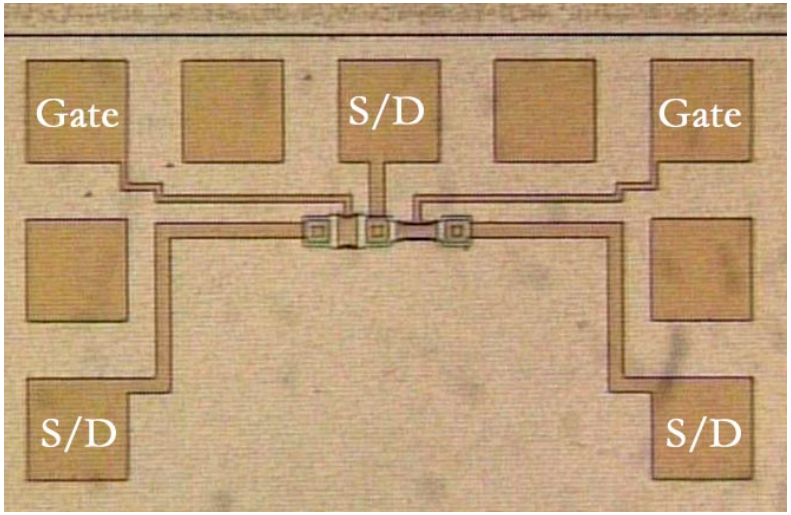
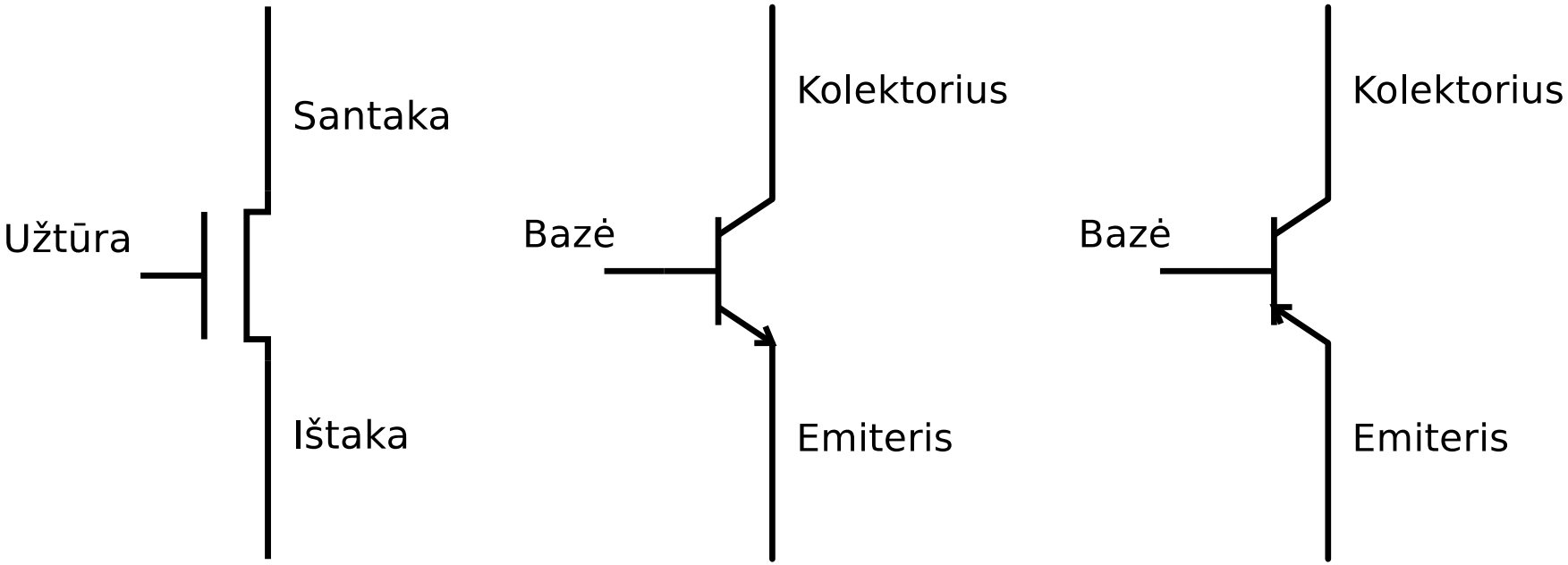


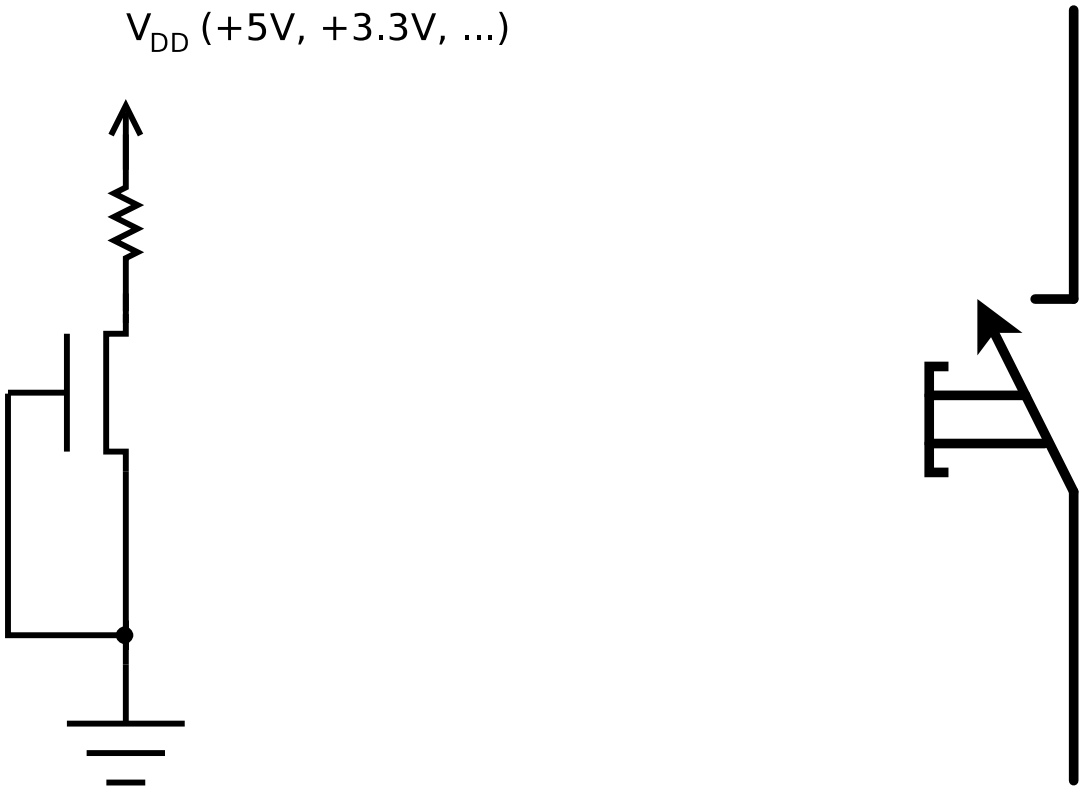


Dick Lyon [Public domain]

Lauko ir bipoliariniai tranzistoriai



Tranzistoriaus savybės



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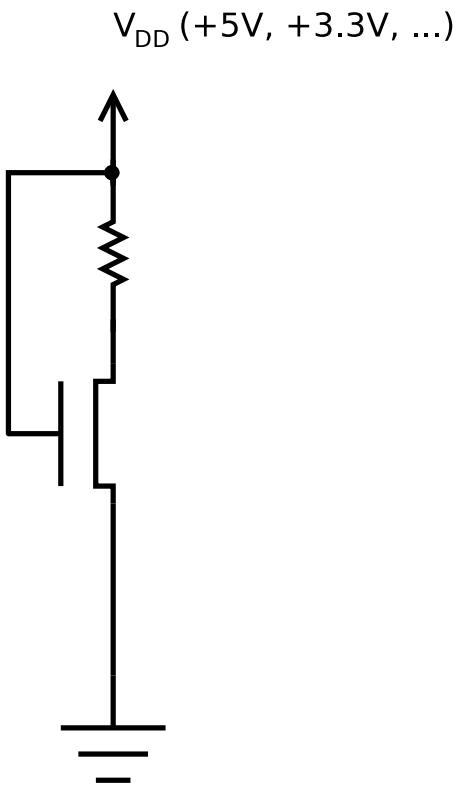
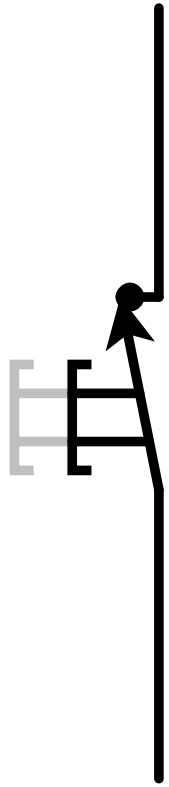
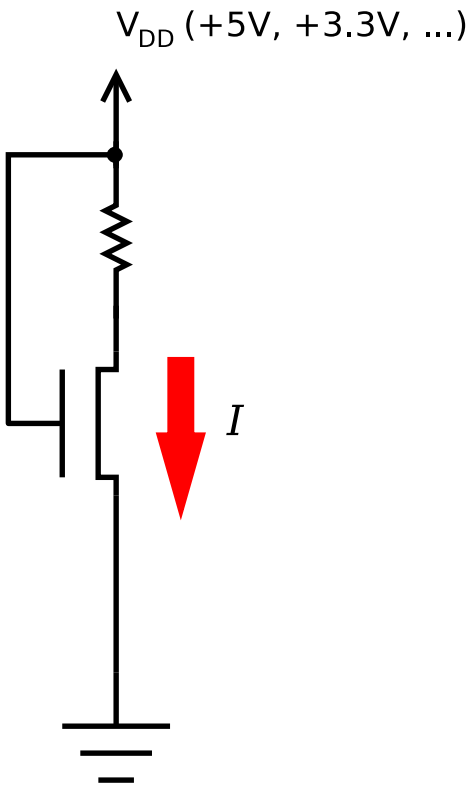
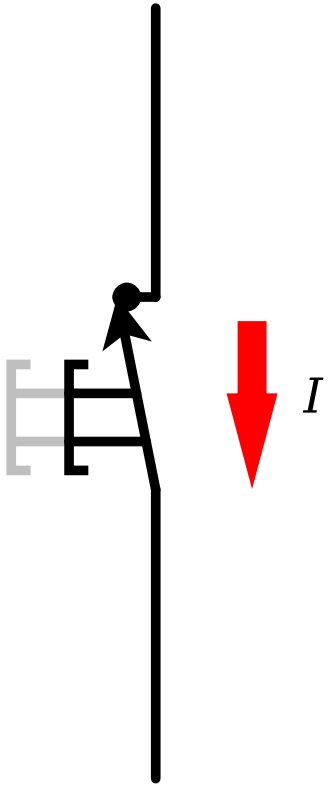


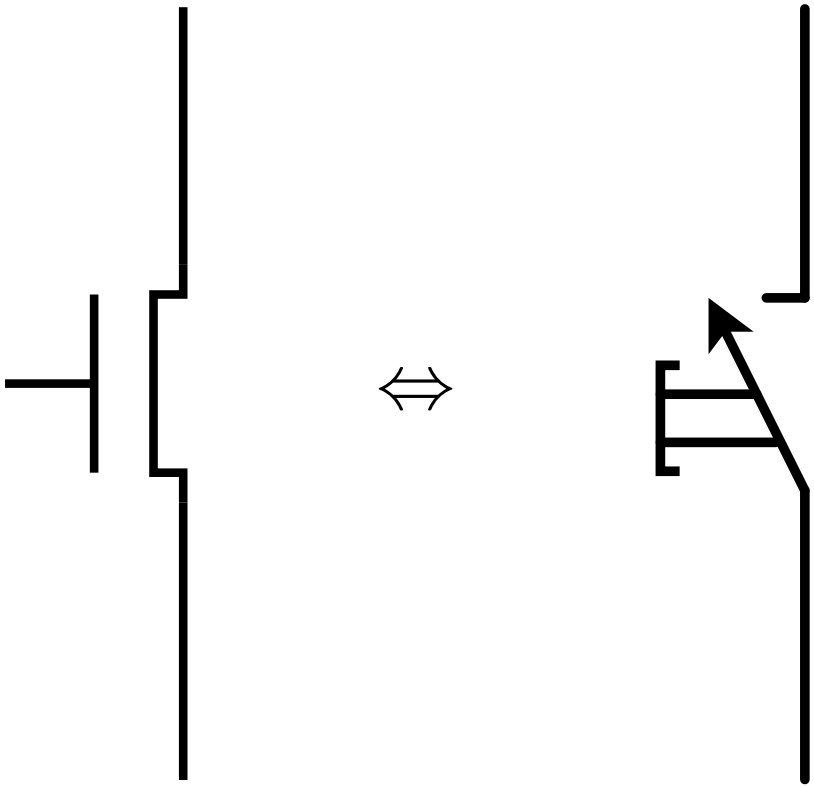


Tranzistoriaus savybės

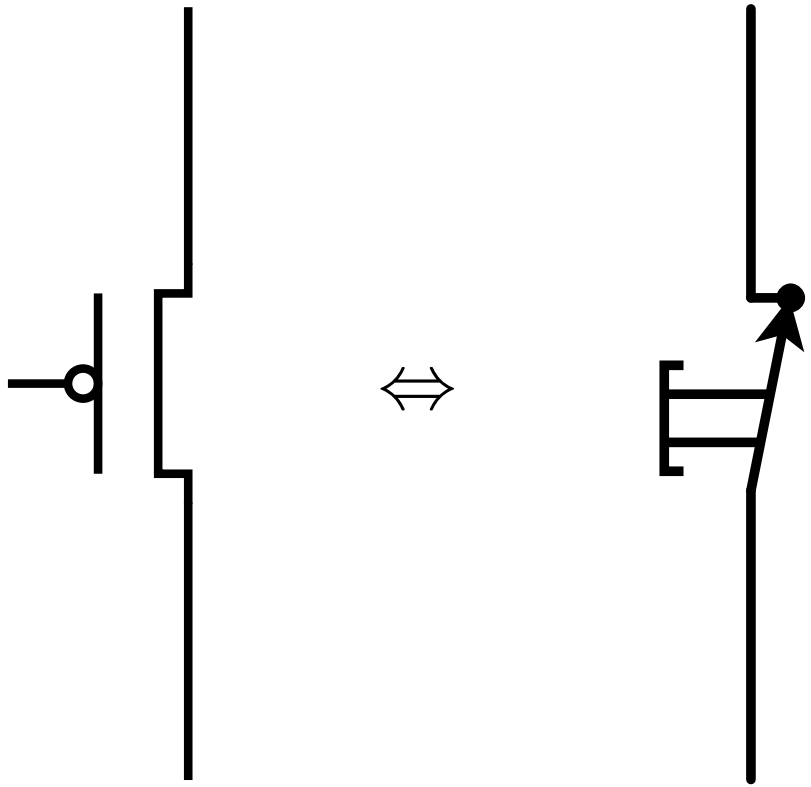




Du tranzistorių tipai

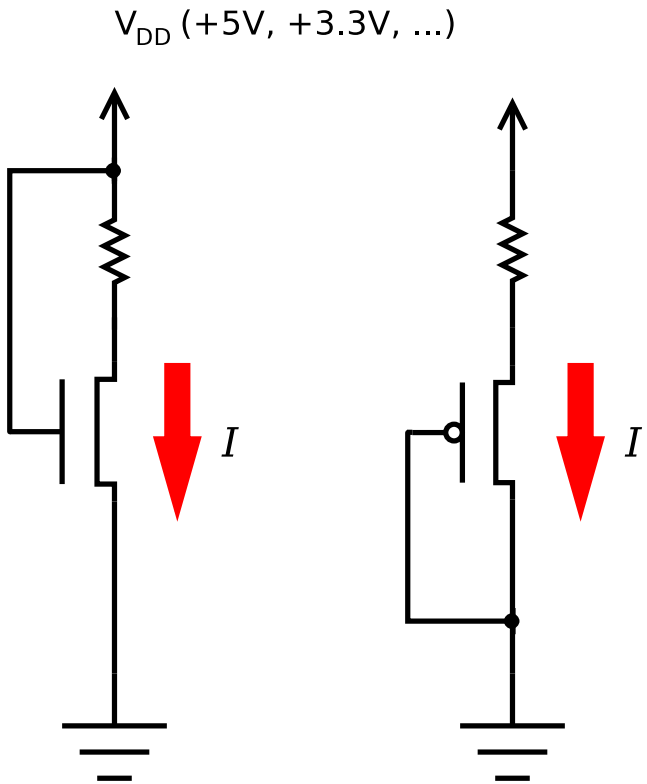


n-tipo tranzistorius

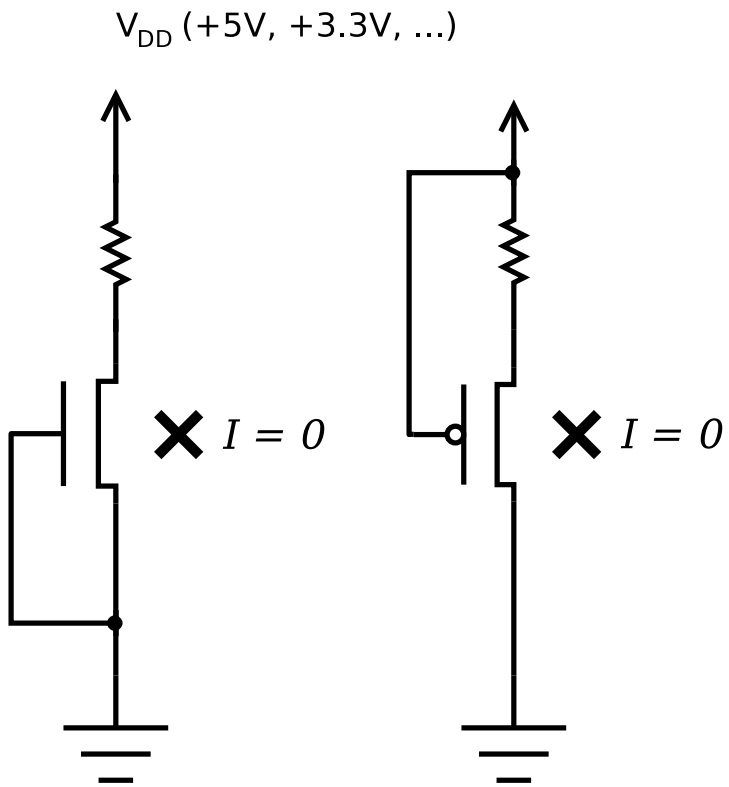


p-tipo tranzistorius

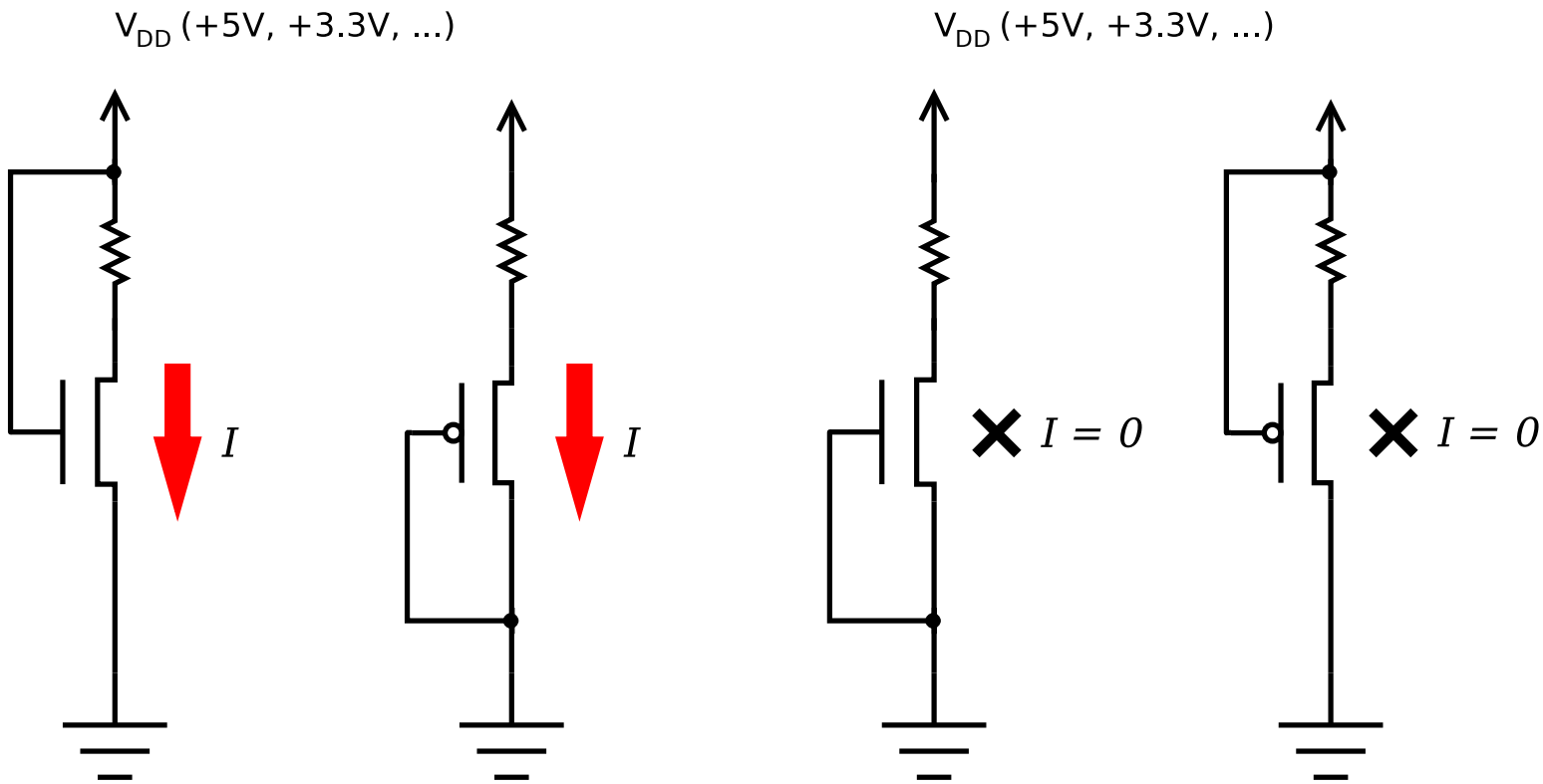
N-tipo ir p-tipo tranzistorių palyginimas



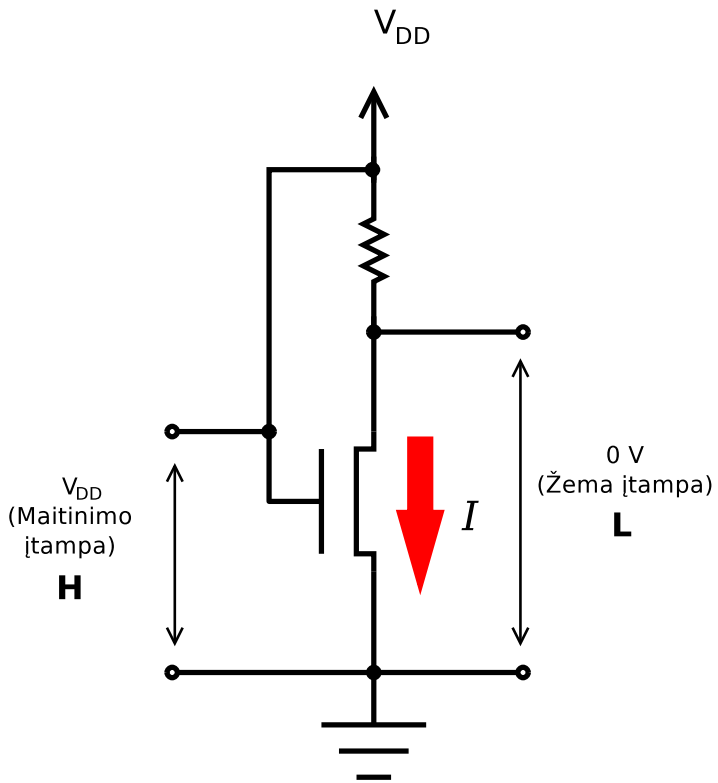
N-tipo ir p-tipo tranzistorių palyginimas

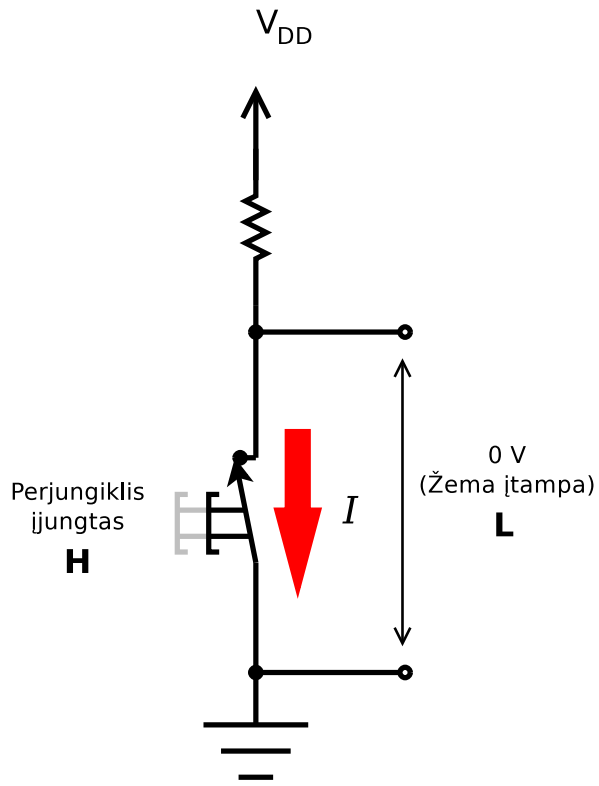


N-tipo ir p-tipo tranzistorių apžvalga

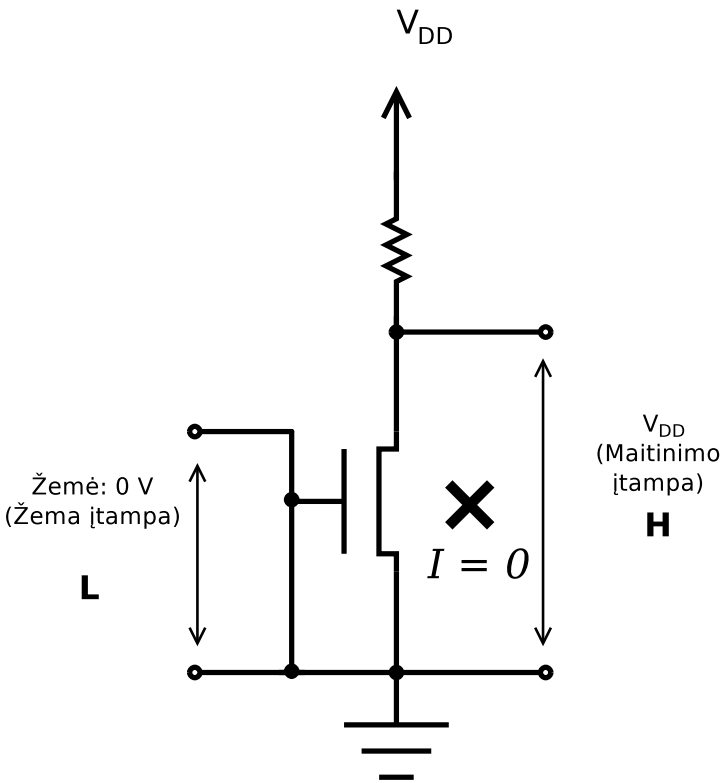


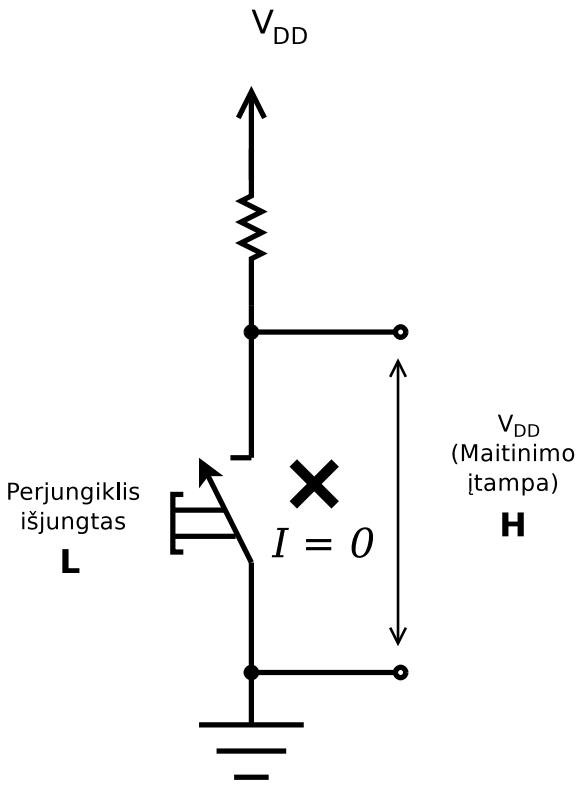
Tranzistorinių perjungiklių įtampos



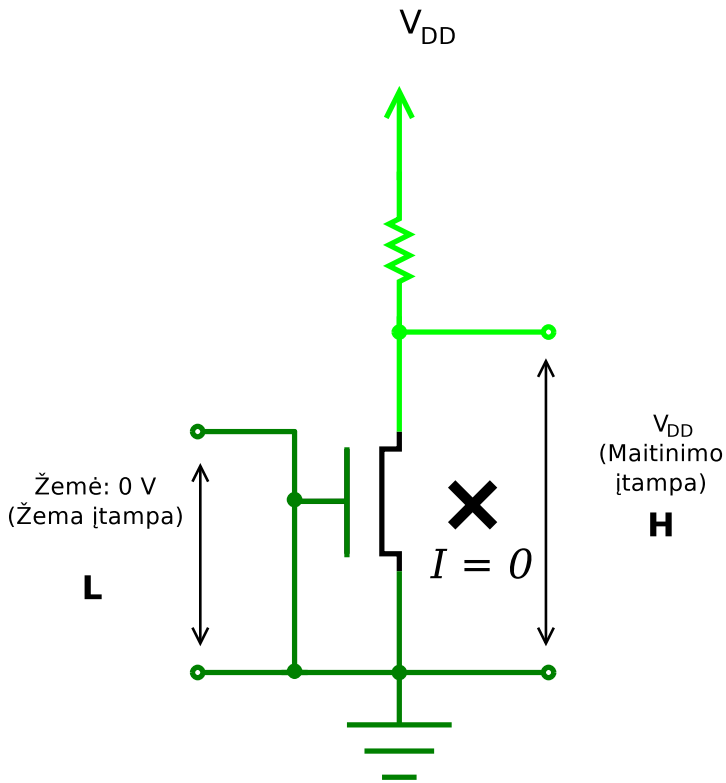


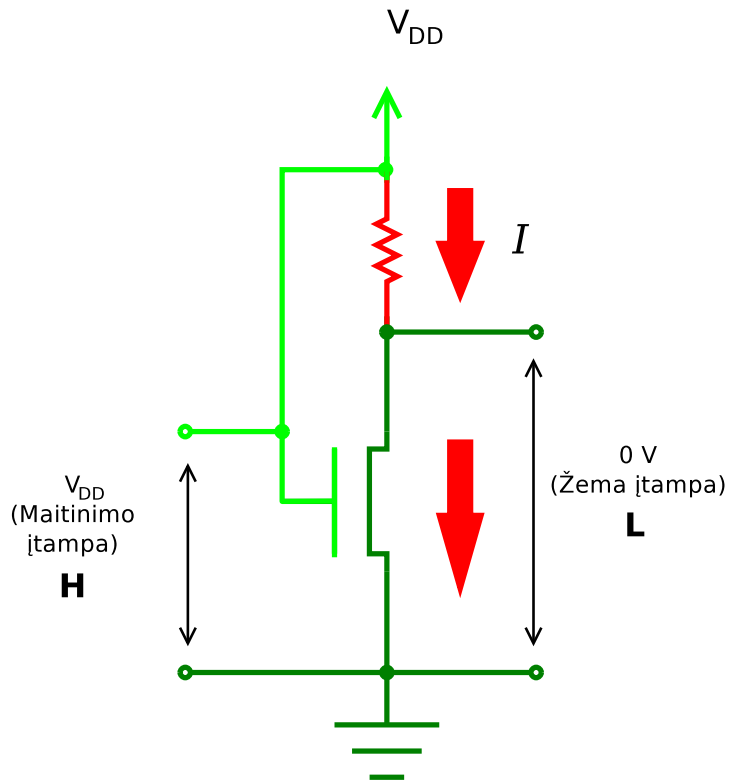
Tranzistorinių perjungiklių įtampos





Priešingos tranzistorinio perjungiklio įtampos





Invertorius (loginė f-ja NE/NOT)

L: Žemės potencialas ($\approx$ 0V)
H: Maitinimo įtampa (5V, 3.3V, ...)

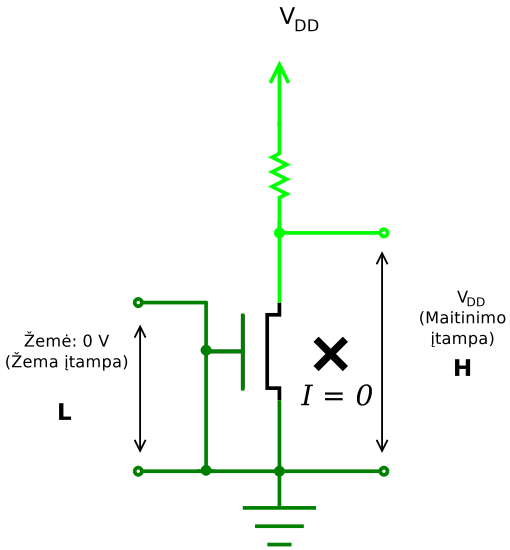
L: 0
H: 1

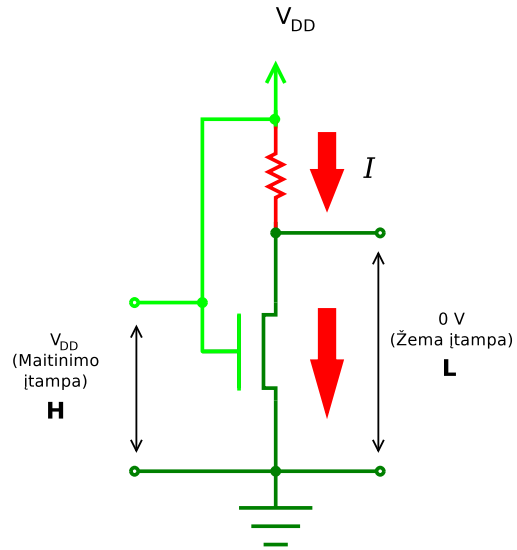
L: False
H: True

Įvestis	Išvestis
L	H
H	L

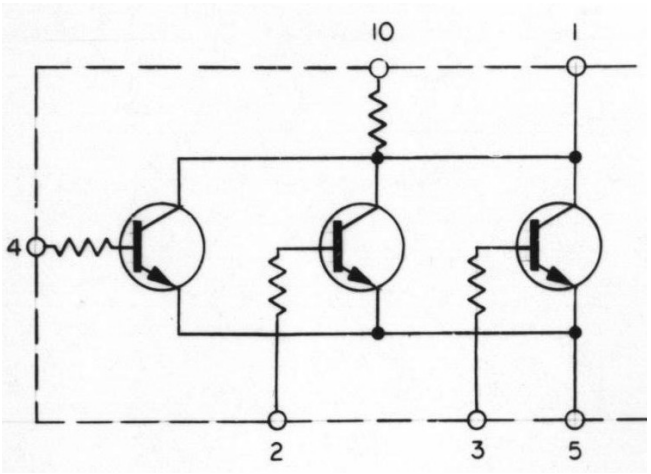
Įvestis	Išvestis
0	1
1	0

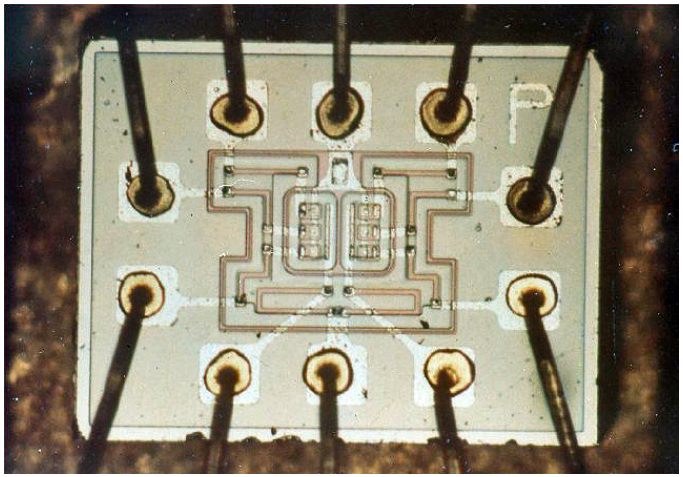
Įvestis	Išvestis
False	True
True	False

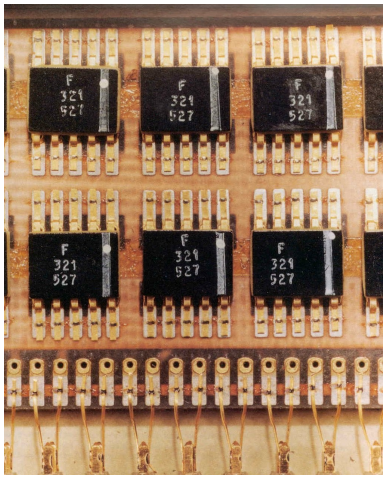


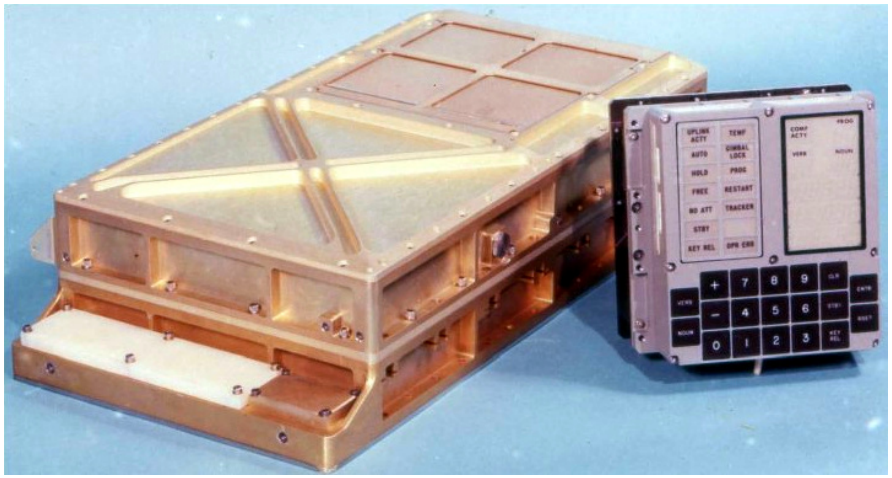


Istorinės loginės schemos: Apolono valdymo kompiuteris



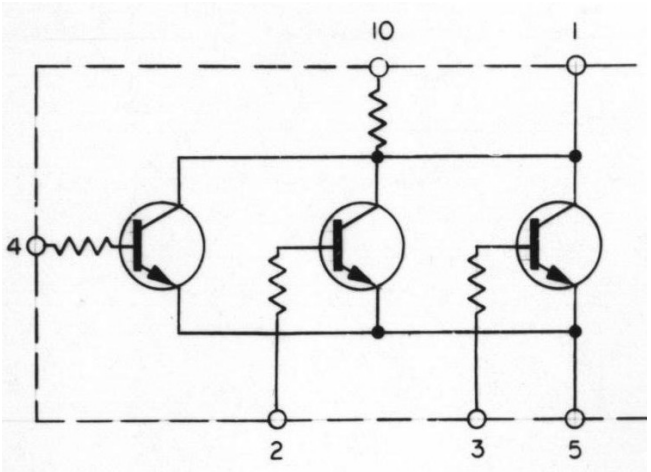


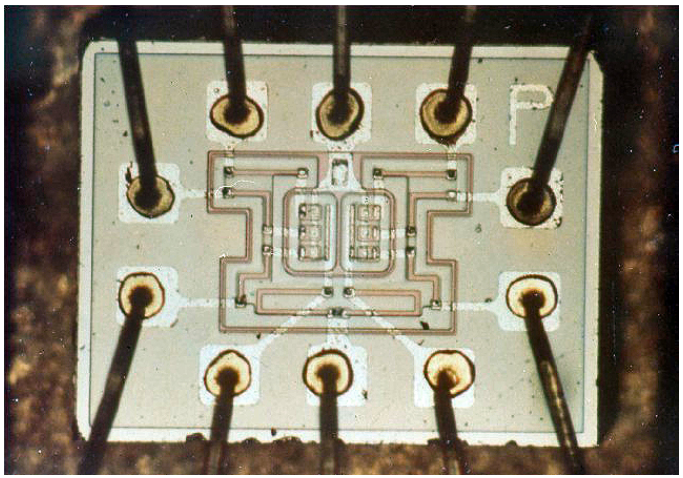


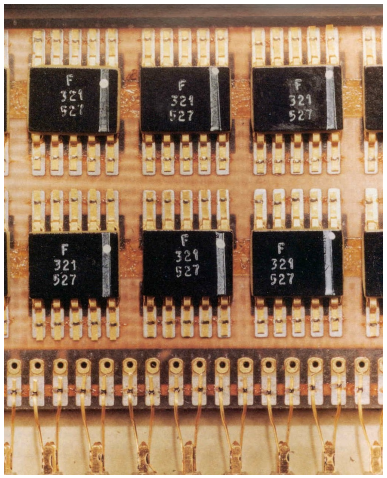


All AGC images: Wikipedia. Apollo guidance computer, (viewed 2020-08-10). All images by NASA, public domain.

Istorinės loginės schemos: Apolono valdymo kompiuteris

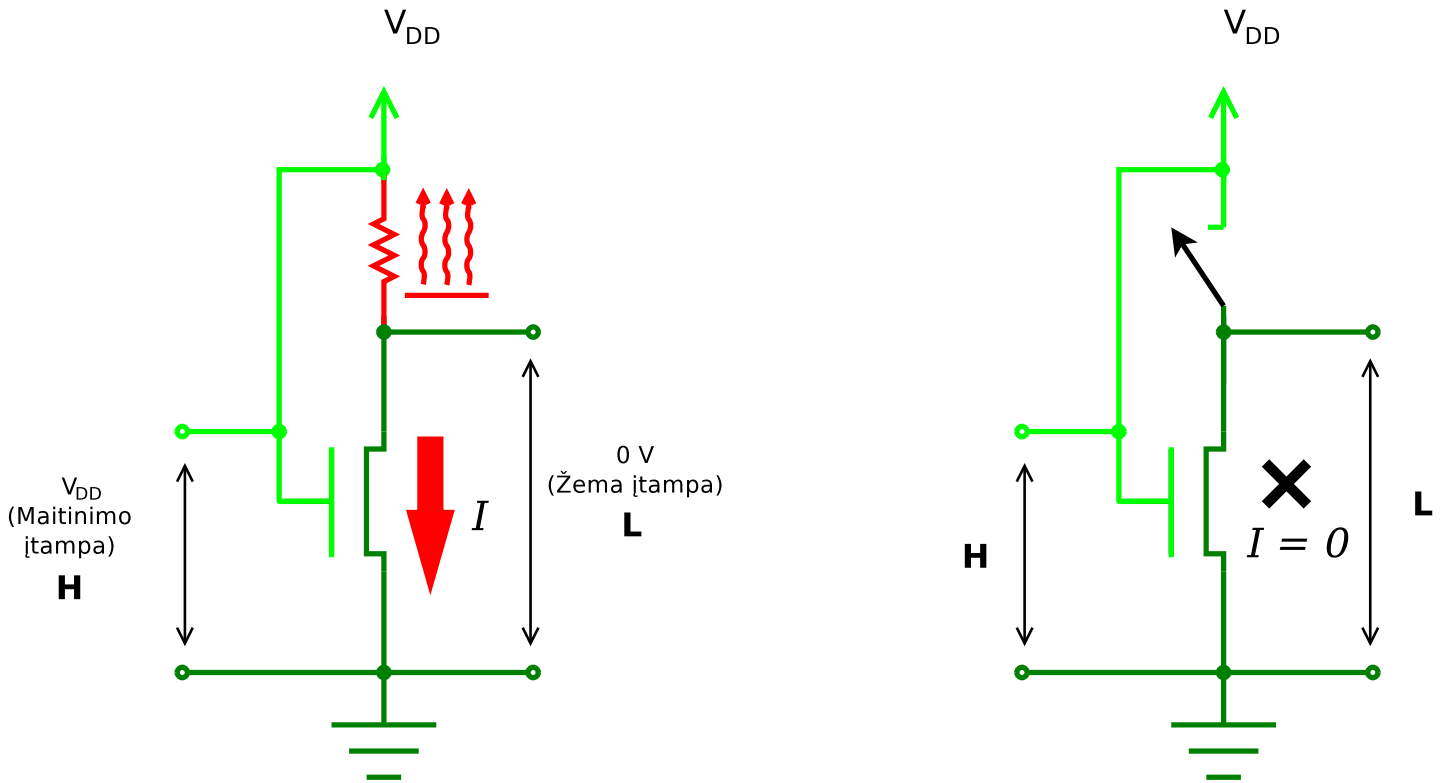




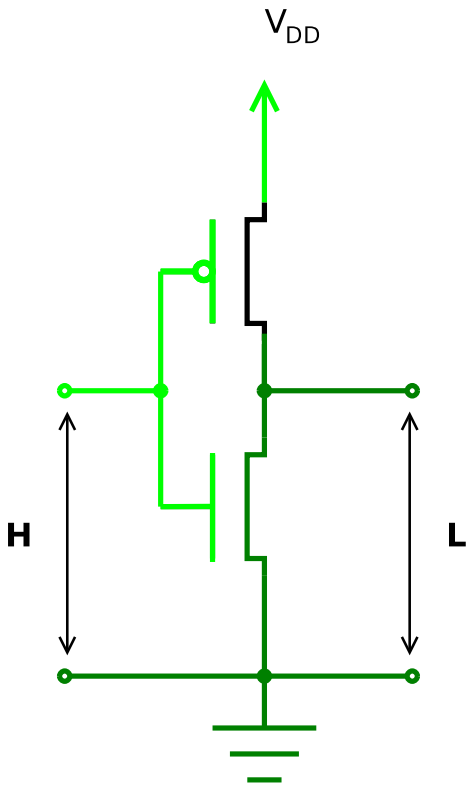


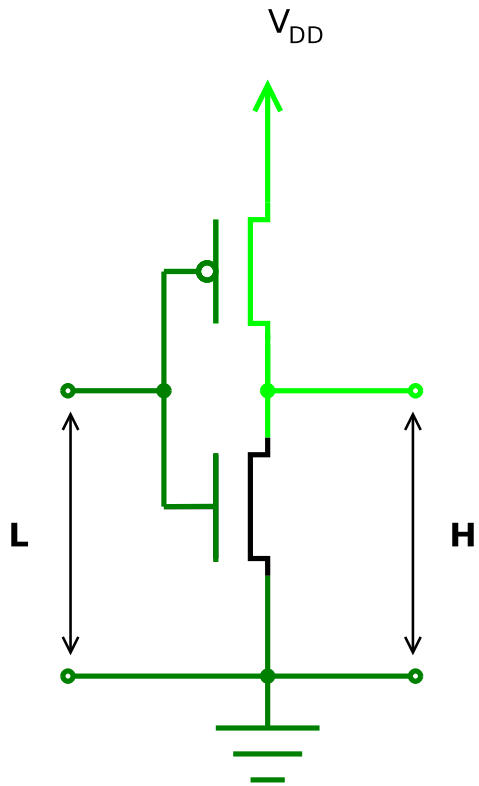
Lunar Landing Module image: by Neil Armstrong (viewed 2020-08-10). [Public domain]

Išsklaidyta galia



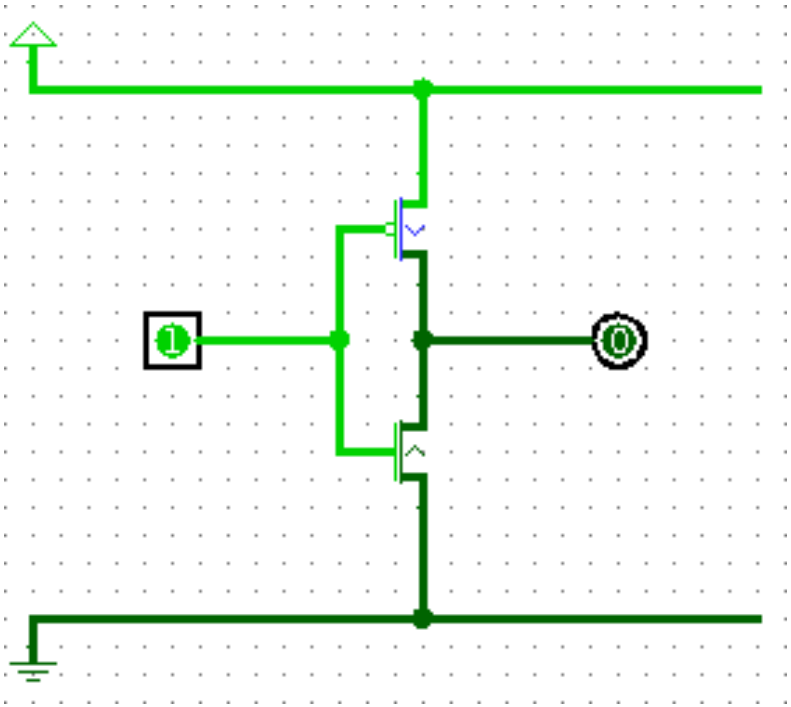
CMOS ventilis NE/NOT

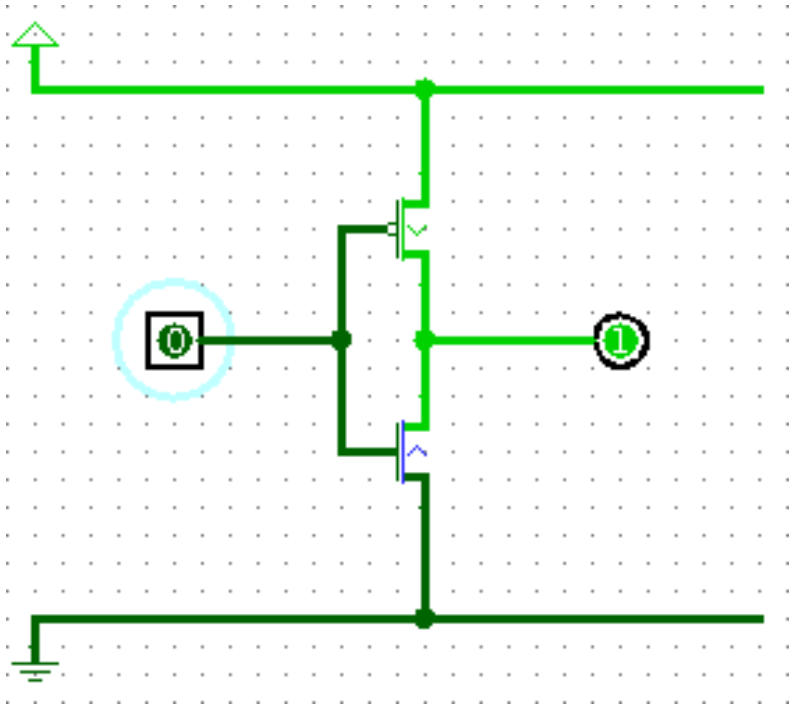




NOT ventilis, sumodeliuotas Logisim

Logisim yra puiki programa skaitmeninių įrenginių konstravimui ir modeliavimui.



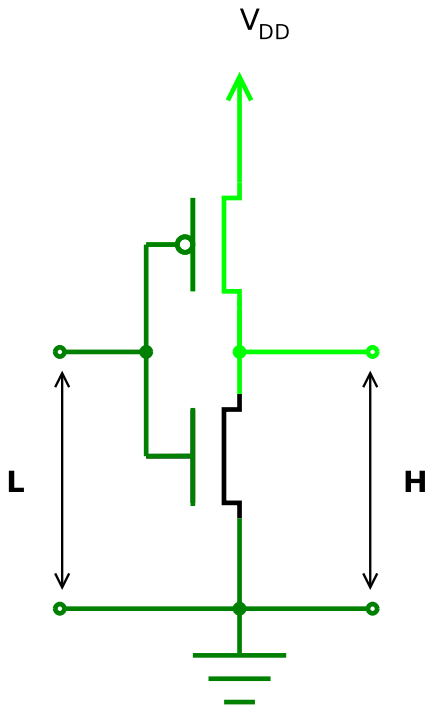


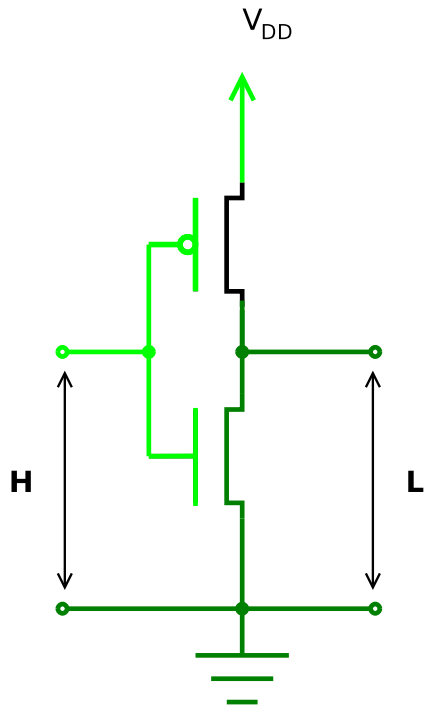
Loginis elementas NE – priminimas

Įvestis	Išvestis
L	H
H	L

Įvestis	Išvestis
0	1
1	0

Įvestis	Išvestis
False	True
True	False





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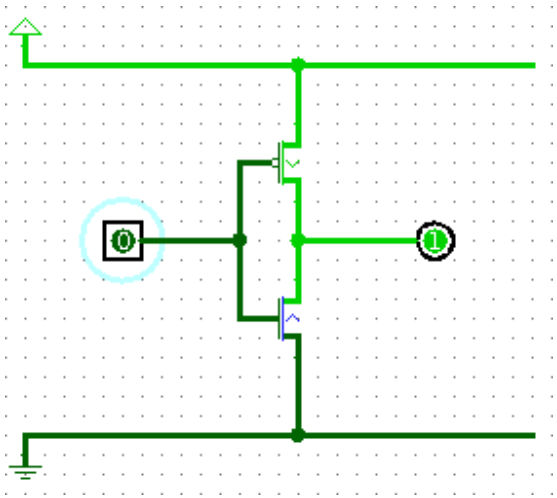

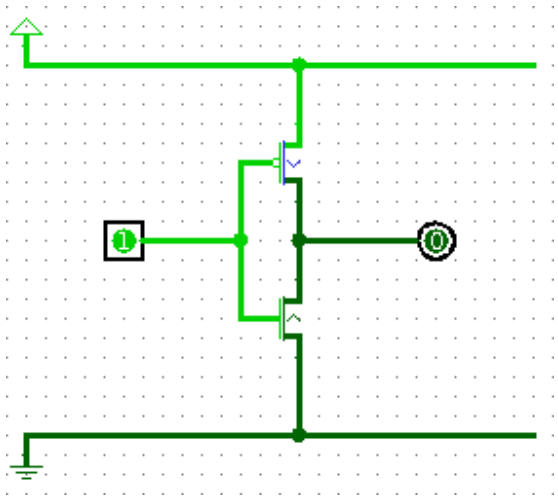

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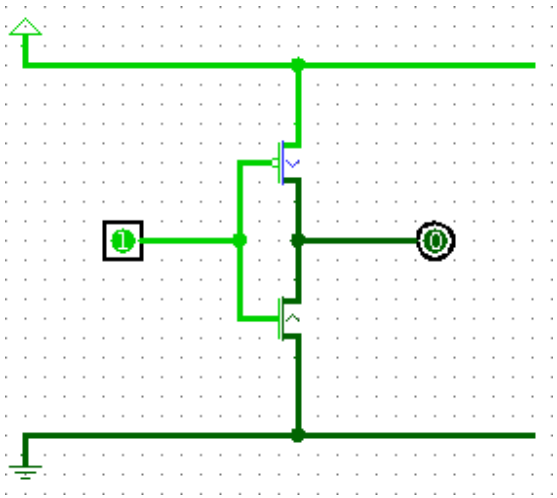
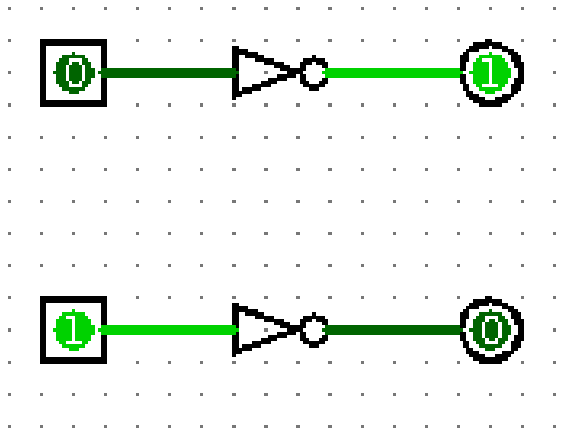


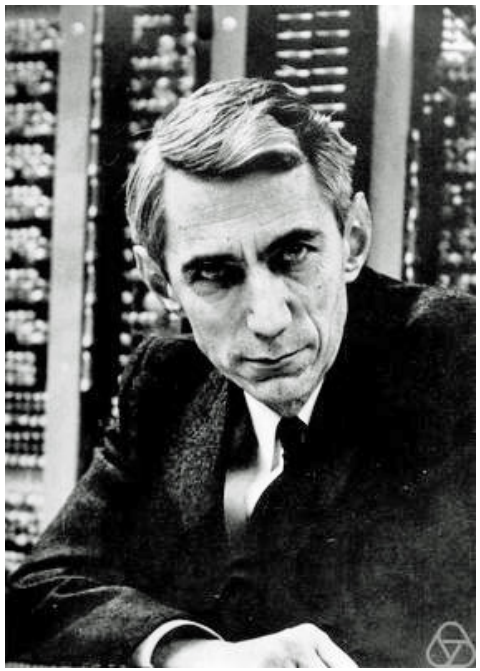


Būlio funkcijos

Paremtos dviejų elementų Būlio algebra, arba „perjungiklių/relių algebra“.

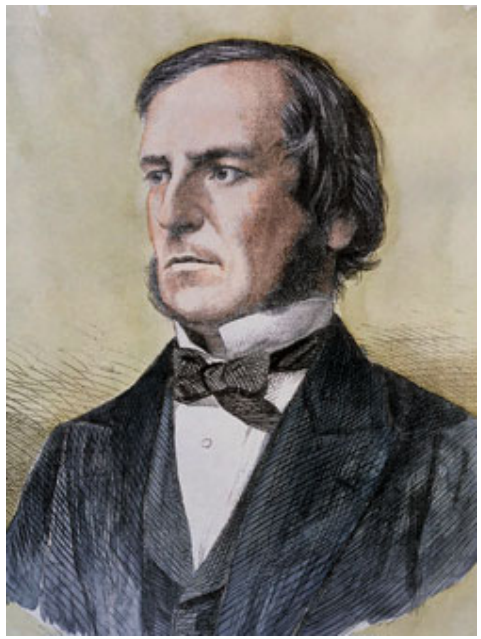
$$B = \{0, 1\}$$
$$B \times \cdots \times B \mapsto B$$

Augustus De Morgan



Claude Shannon

Photo by Konrad Jacobs, reused with permission



George Boole

Vieno argumento funkcijų skaičius

Įvestis	Išvestis
0	1
1	0

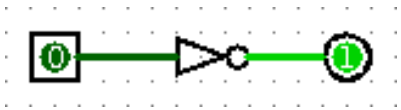
2 positions $\Rightarrow 2^2 = 4$ combinations

Įvestis	Išvestis
0	0
1	1

identity $f(x) = x$

Įvestis	Išvestis
0	1
1	0

NOT $f(x) = \overline{x}$

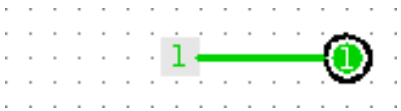


Įvestis	Išvestis
0	0
1	0

constant 0 $f(x) = 0$

Įvestis	Išvestis
0	1
1	1

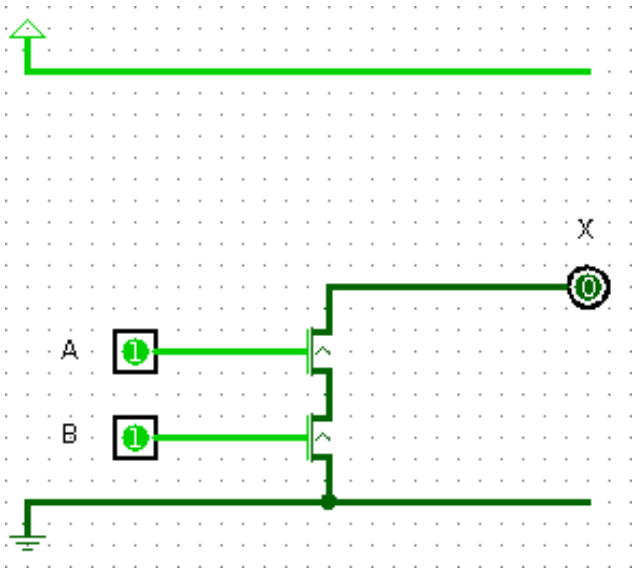
constant 1 $f(x) = 1$



Loginė funkcija IR-NE

$$X = f(A, B) = \overline{A \cdot B} = \overline{AB}$$

A	B	X
0	0	1
0	1	1
1	0	1
1	1	0



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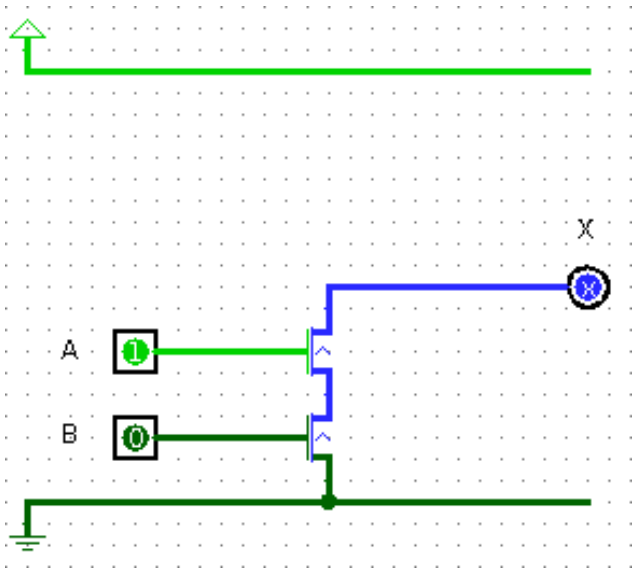
0	1	1
1	0	1
1	1	0



Loginė funkcija IR-NE

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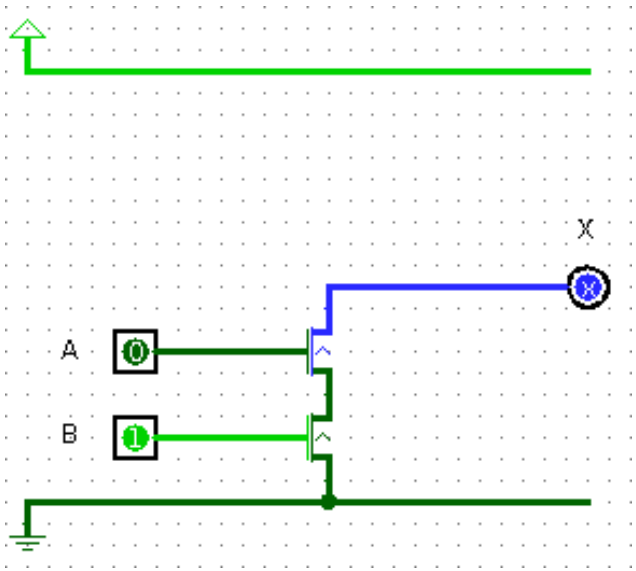
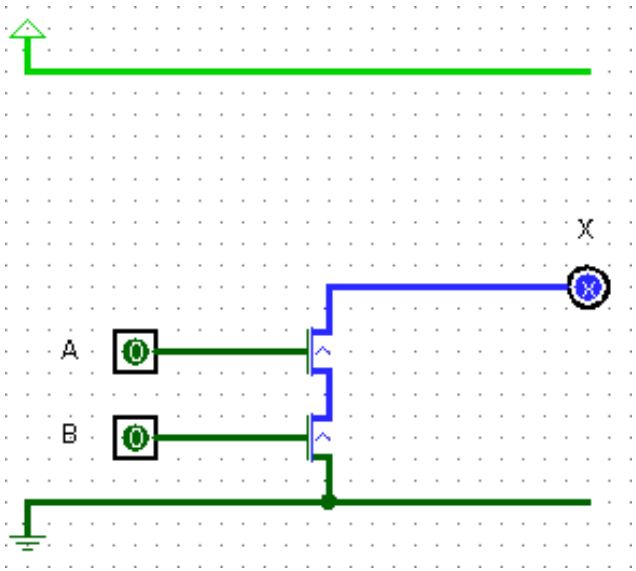
0	1	1
1	0	1
1	1	0



Loginė funkcija IR-NE

$$X = f(A, B) = \overline{A \cdot B} = \overline{AB}$$

A	B	X
0	0	1
0	1	1
1	0	1
1	1	0



Loginė funkcija IR-NE

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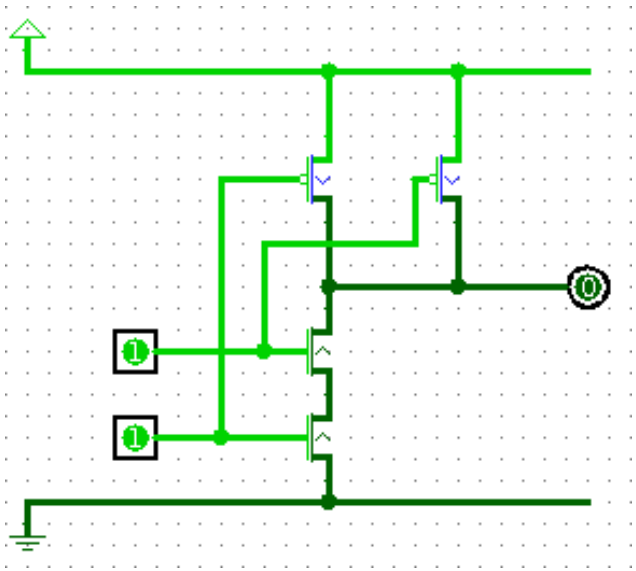
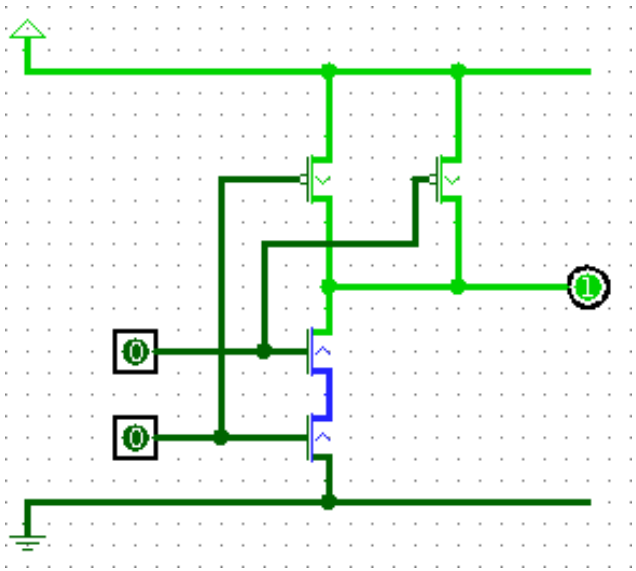
A	B	X
0	0	1
0	1	1
1	0	1
1	1	0



Loginė funkcija IR-NE

$$X = f(A, B) = \overline{A \cdot B} = \overline{AB}$$

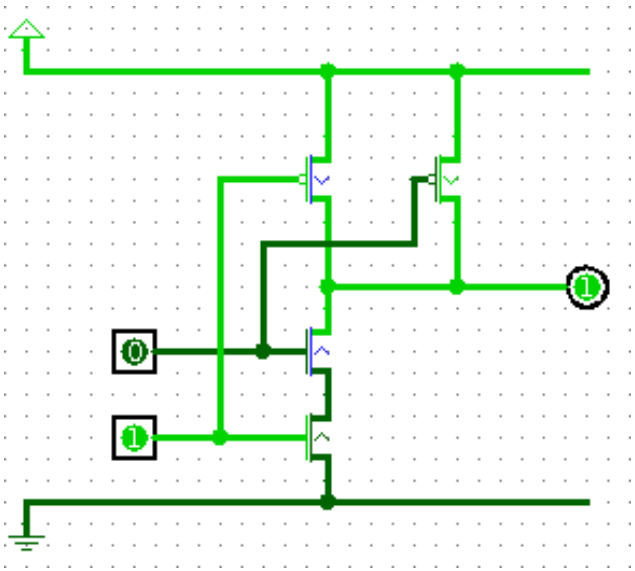
A	**B**	**X**
0	**0**	**1**
0	1	1
1	0	1
1	1	0



Loginė funkcija IR-NE

$$X = f(A, B) = \overline{A \cdot B} = \overline{AB}$$

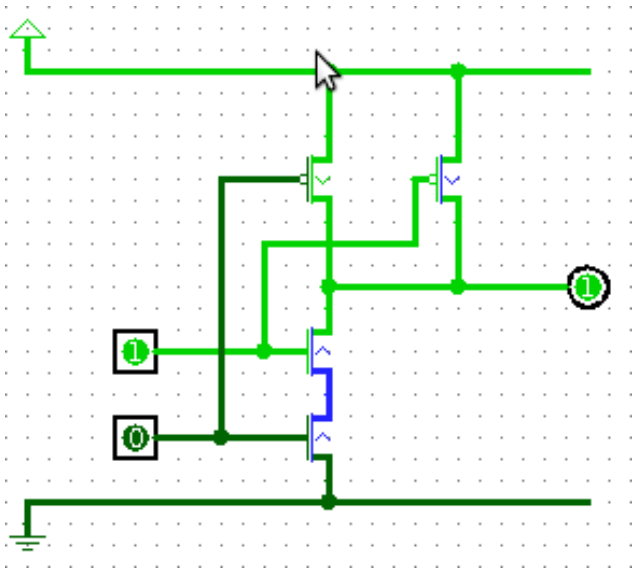
A	**B**	**X**
0	0	1
0	**1**	**1**
1	0	1
1	1	0



Loginė funkcija IR-NE

$$X = f(A, B) = \overline{A \cdot B} = \overline{AB}$$

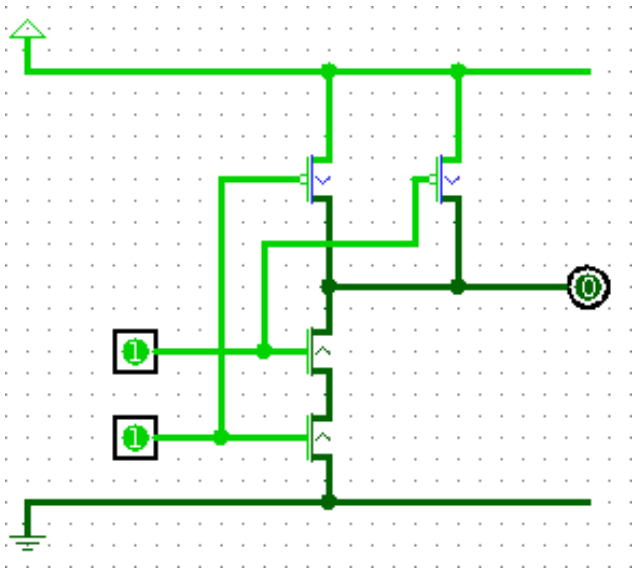
A	B	X
0	0	1
0	1	1
1	**0**	**1**
1	1	0



Loginė funkcija IR-NE

$$X = f(A, B) = \overline{A \cdot B} = \overline{AB}$$

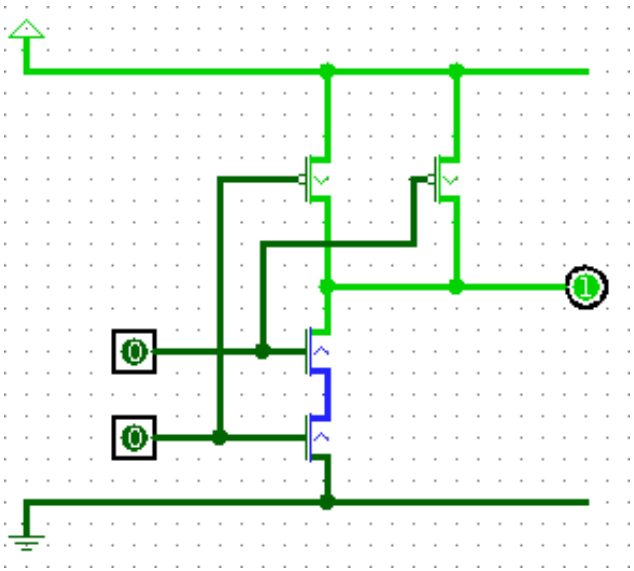
A	B	X
0	0	1
0	1	1
1	0	1
1	**1**	**0**

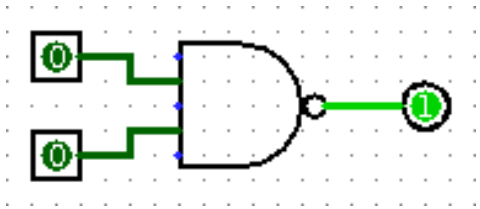


Loginė funkcija IR-NE

$$X = f(A, B) = \overline{A \cdot B} = \overline{AB}$$

A	B	X
0	0	1
0	1	1
1	0	1
1	1	0

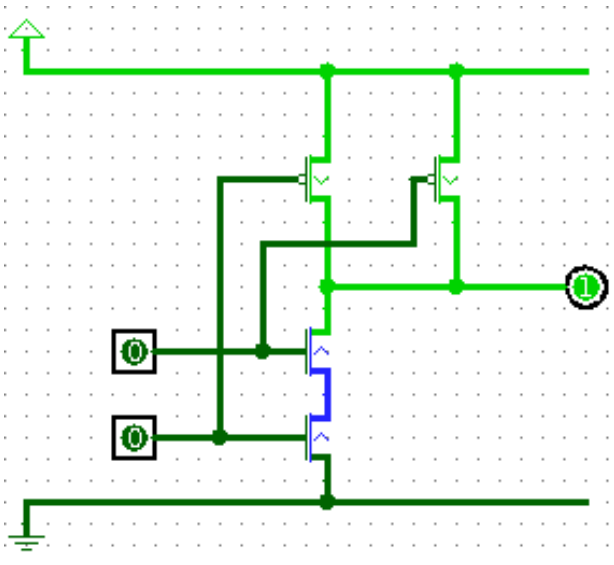


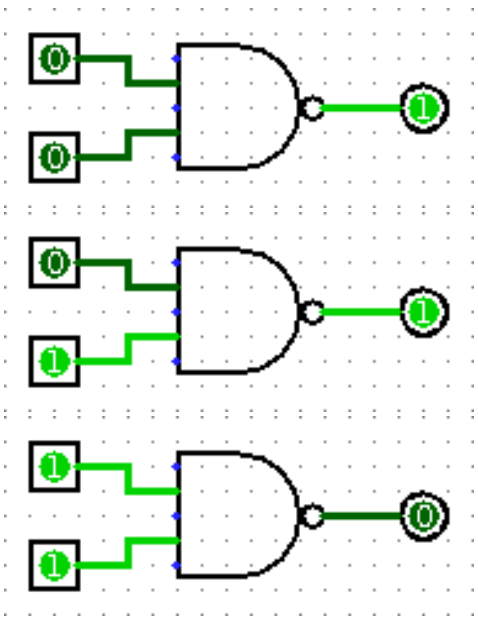


Loginė funkcija IR-NE

$$X = f(A, B) = \overline{A \cdot B} = \overline{AB}$$

A	B	X
0	0	1
0	1	1
1	0	1
1	1	0

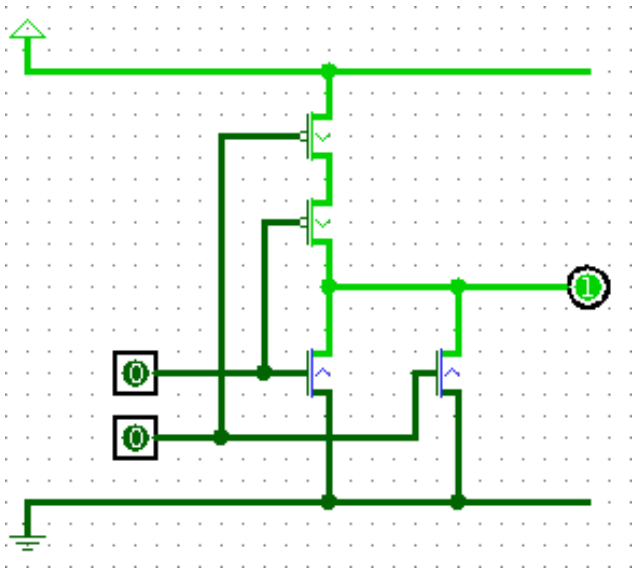




Loginė funkcija ARBA-NE

$$X = f(A, B) = \overline{A \vee B}$$

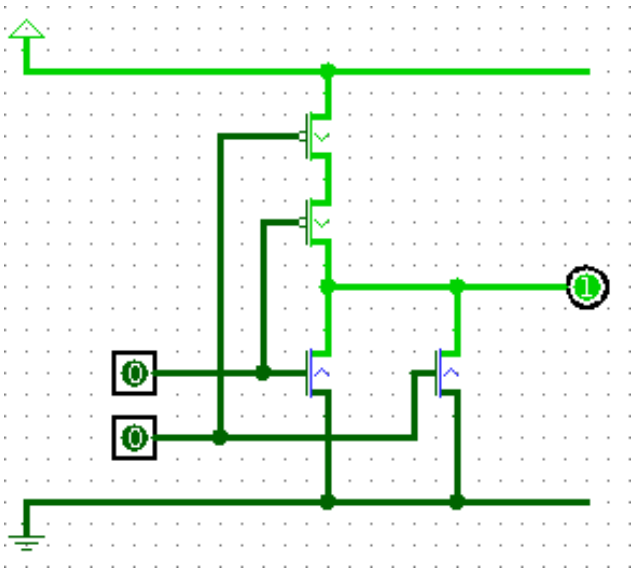
A	B	X
0	0	1
0	1	0
1	0	0
1	1	0



Loginė funkcija ARBA-NE

$$X = f(A, B) = \overline{A \vee B}$$

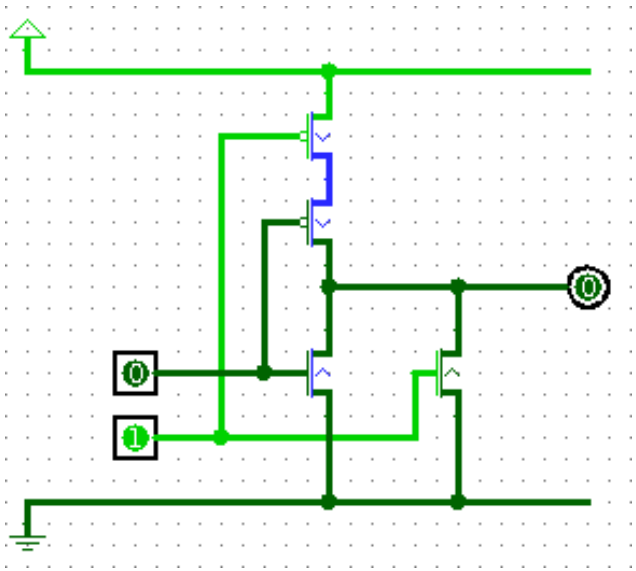
A	**B**	**X**
0	**0**	**1**
0	1	0
1	0	0
1	1	0



Loginė funkcija ARBA-NE

$$X = f(A, B) = \overline{A \vee B}$$

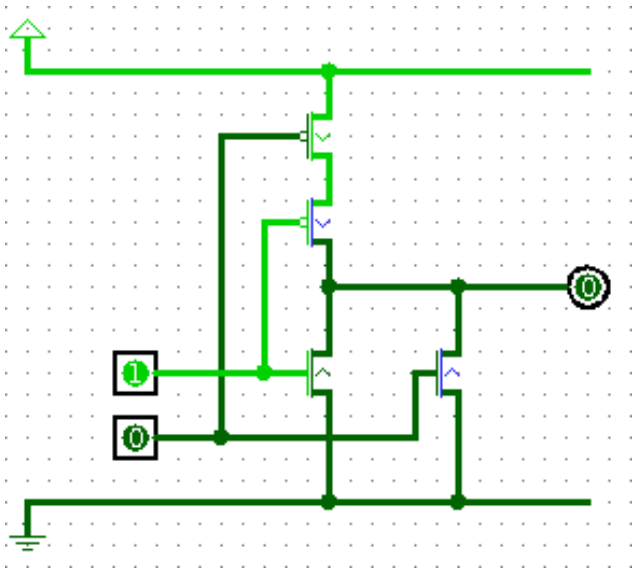
A	**B**	**X**
0	0	1
0	**1**	**0**
1	0	0
1	1	0



Loginė funkcija ARBA-NE

$$X = f(A, B) = \overline{A \vee B}$$

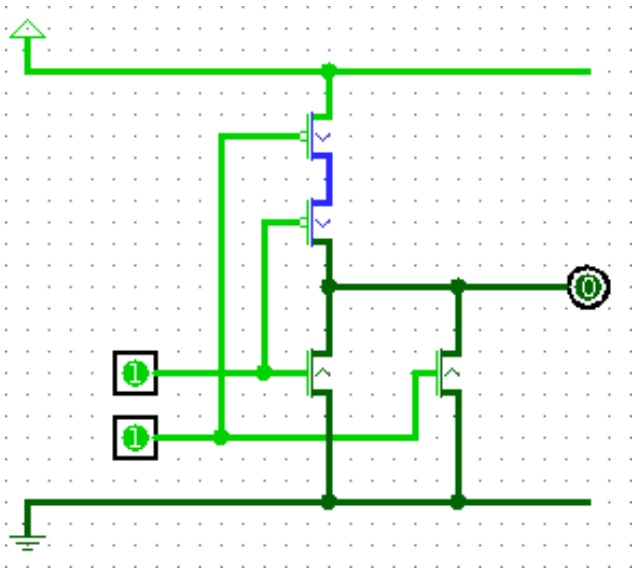
A	B	X
0	0	1
0	1	0
1	**0**	**0**
1	1	0



Loginė funkcija ARBA-NE

$$X = f(A, B) = \overline{A \vee B}$$

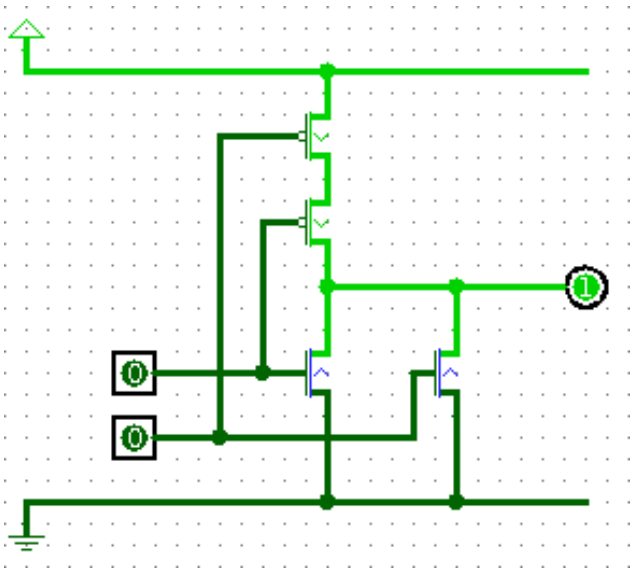
A	B	X
0	0	1
0	1	0
1	0	0
1	**1**	**0**

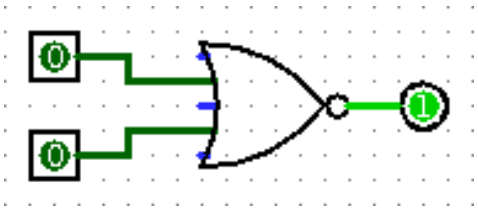


Loginė funkcija ARBA-NE

$$X = f(A, B) = \overline{A \vee B}$$

A	B	X
0	0	1
0	1	0
1	0	0
1	1	0

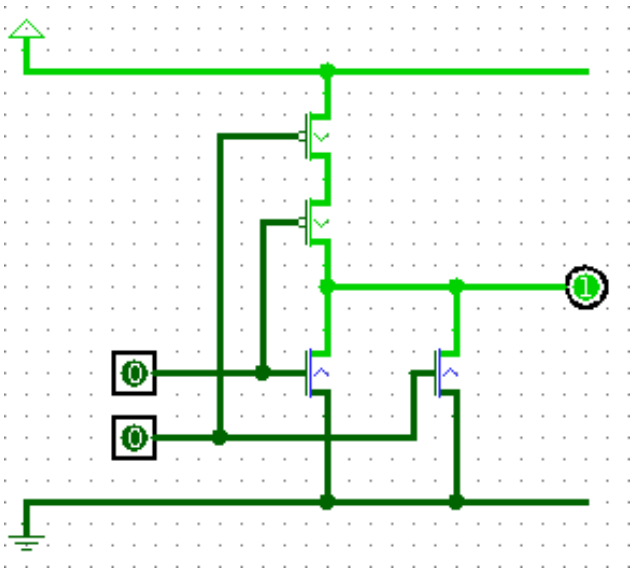


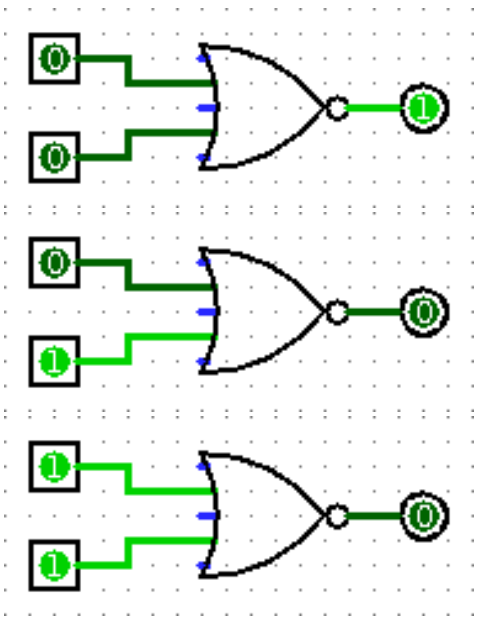


Loginė funkcija ARBA-NE

$$X = f(A, B) = \overline{A \vee B}$$

A	B	X
0	0	1
0	1	0
1	0	0
1	1	0

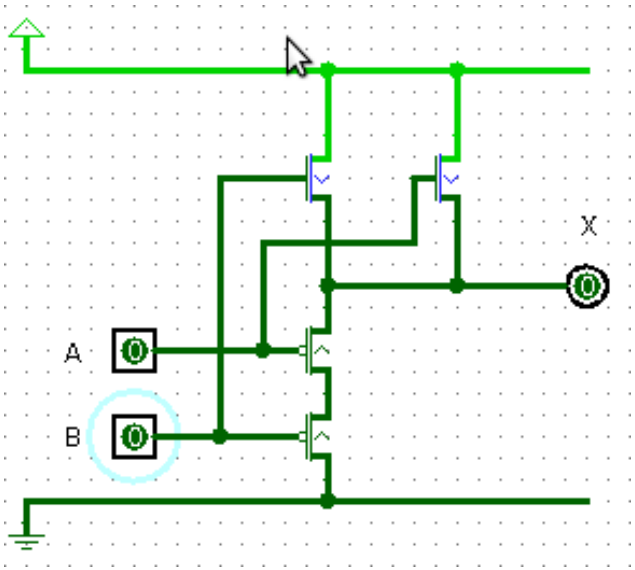




Loginė funkcija ARBA (disjunkcija)

$$X = f(A, B) = A \vee B$$

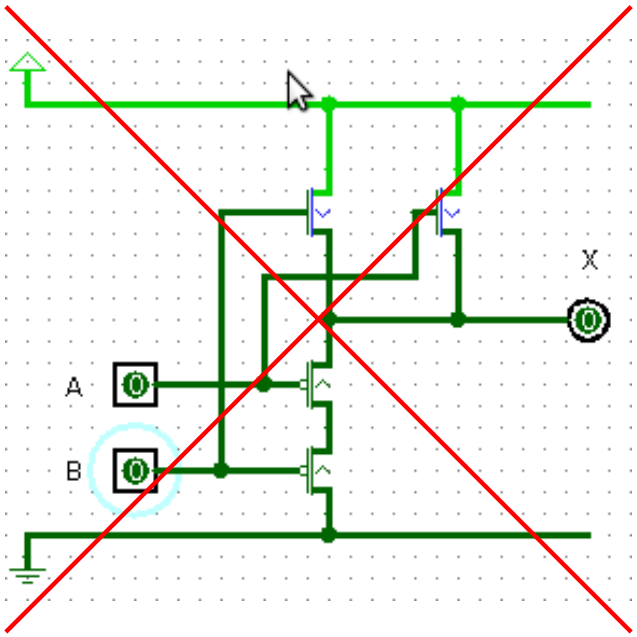
A	B	X
0	0	0
0	1	1
1	0	1
1	1	1



Loginė funkcija ARBA (disjunkcija)

$$X = f(A, B) = A \vee B$$

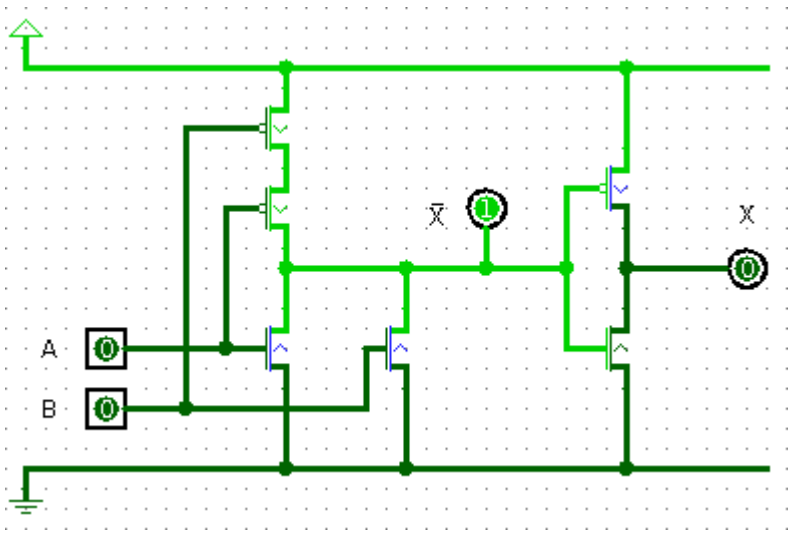
A	B	X
0	0	0
0	1	1
1	0	1
1	1	1



Loginė funkcija ARBA (disjunkcija)

$$X = f(A, B) = A \vee B$$

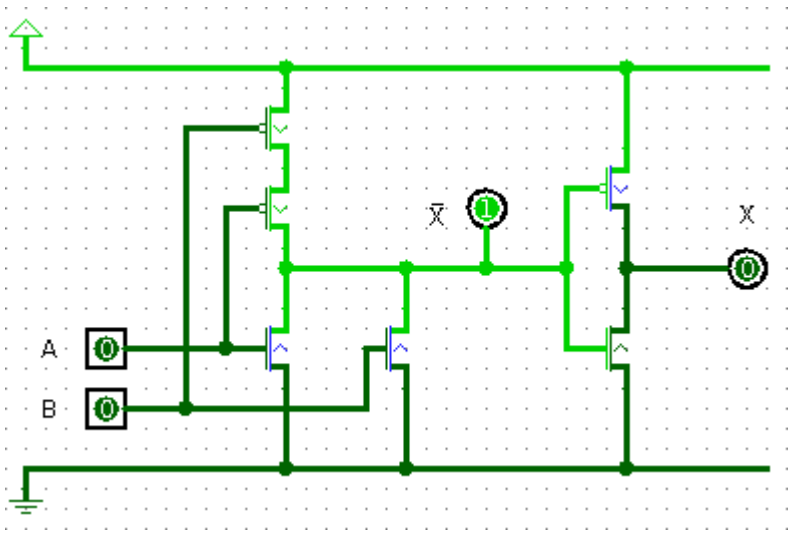
A	B	X
0	0	0
0	1	1
1	0	1
1	1	1



Loginė funkcija ARBA (disjunkcija)

$$X = f(A, B) = A \vee B$$

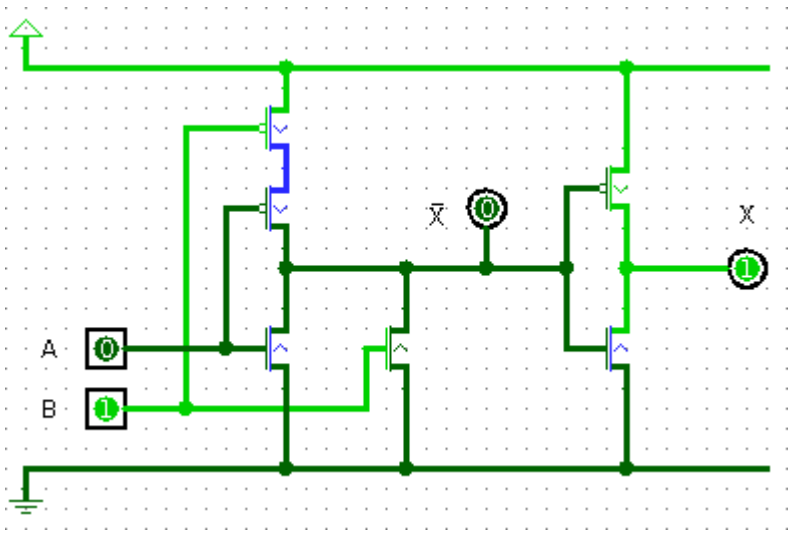
A	**B**	**X**
0	**0**	**0**
0	1	1
1	0	1
1	1	1



Loginė funkcija ARBA (disjunkcija)

$$X = f(A, B) = A \vee B$$

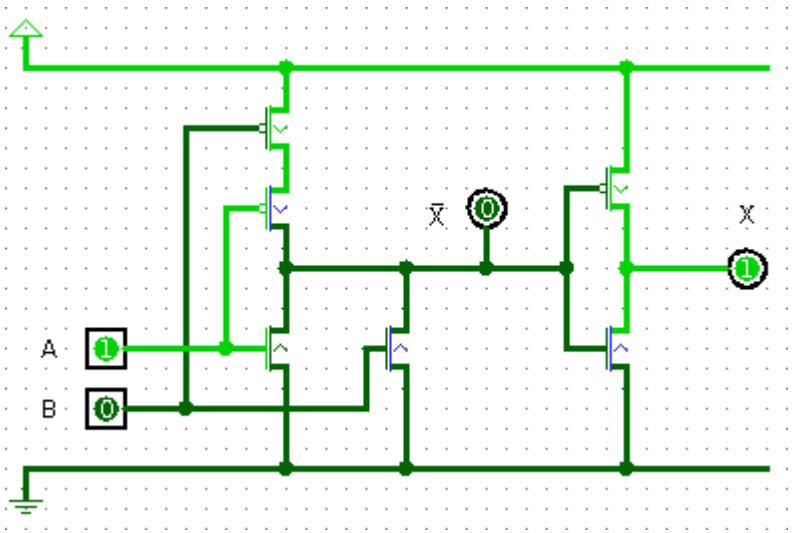
A	**B**	**X**
0	0	0
0	**1**	**1**
1	0	1
1	1	1



Loginė funkcija ARBA (disjunkcija)

$$X = f(A, B) = A \vee B$$

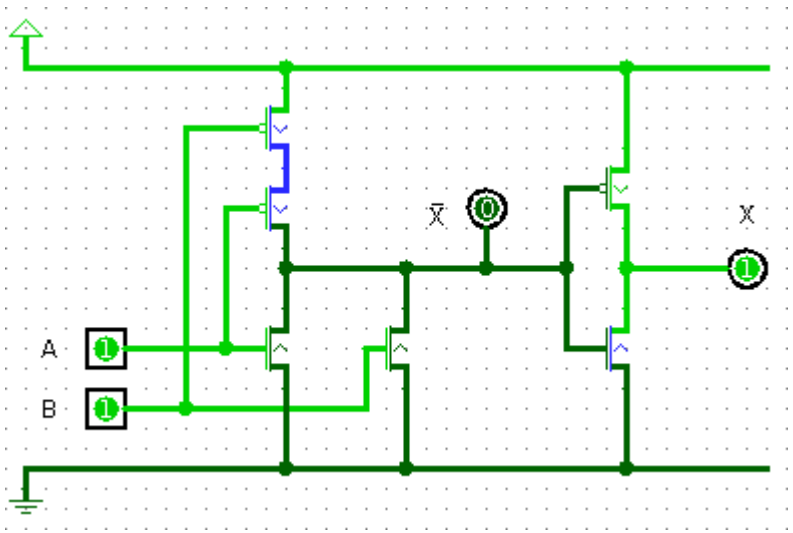
A	**B**	**X**
0	0	0
0	1	1
1	**0**	**1**
1	1	1



Loginė funkcija ARBA (disjunkcija)

$$X = f(A, B) = A \vee B$$

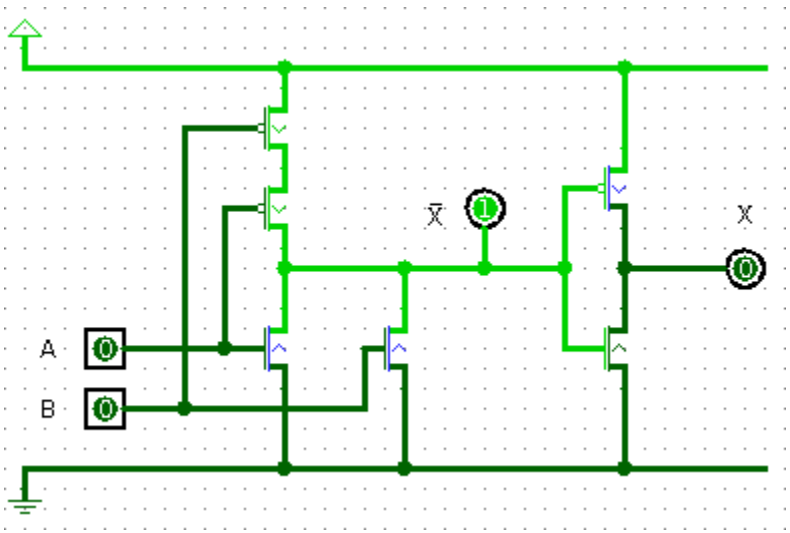
A	B	X
0	0	0
0	1	1
1	0	1
1	**1**	**1**

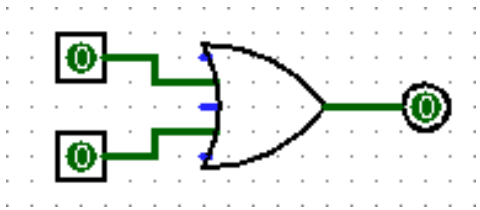


Loginė funkcija ARBA (disjunkcija)

$$X = f(A, B) = A \vee B$$

A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

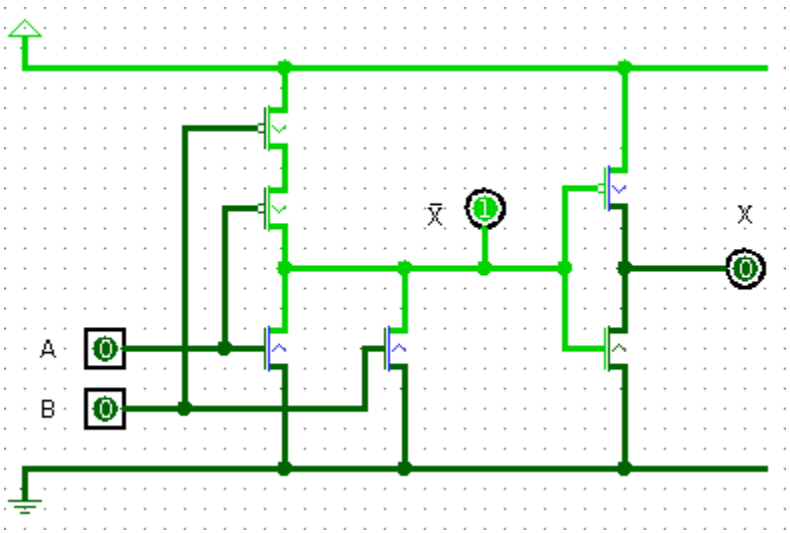


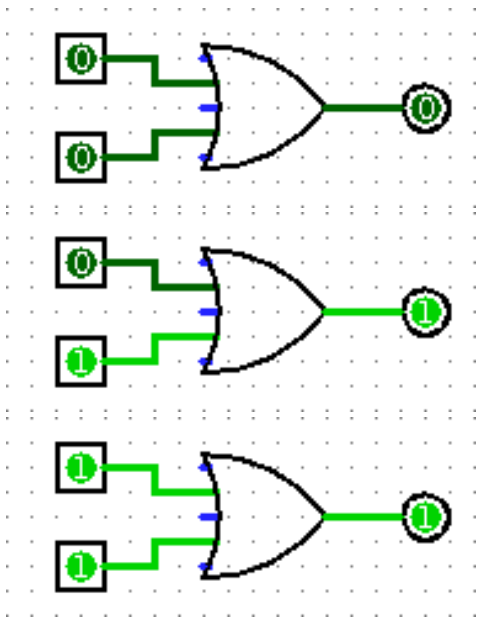


Loginė funkcija ARBA (disjunkcija)

$$X = f(A, B) = A \vee B$$

A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

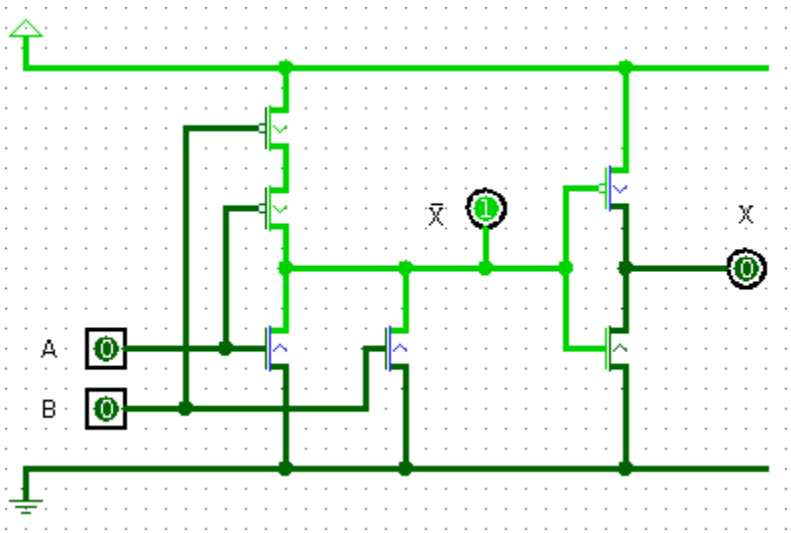


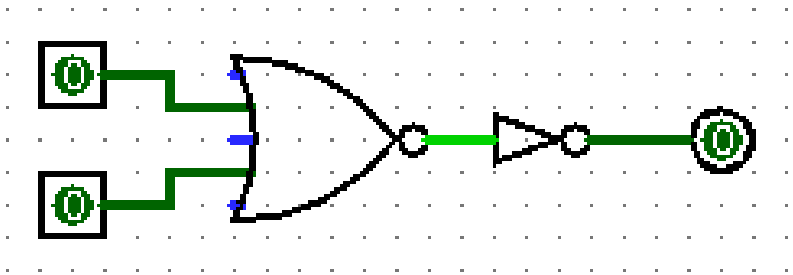


Dviejų argumentų funkcijos

A	B	X
0	0	1
0	1	1
1	0	1
1	1	0

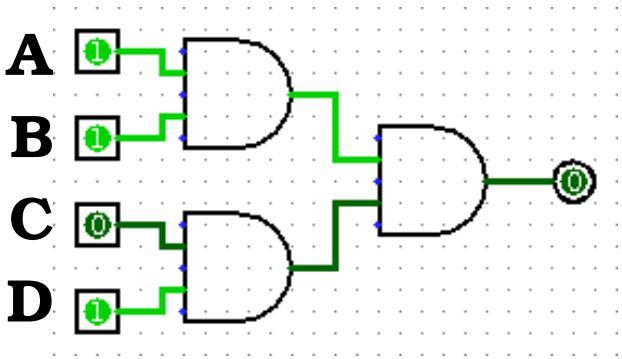
$\Rightarrow 4$ positions $\Rightarrow 2^{2^2} = 16$ combinations

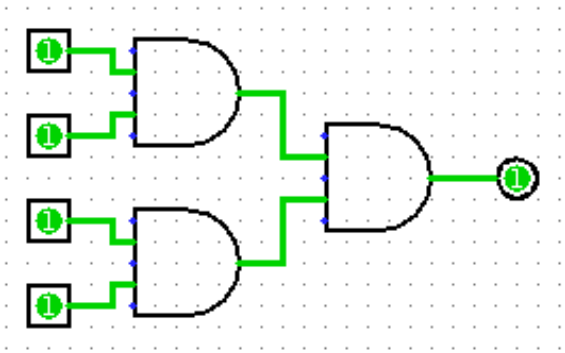




Daugelio įėjimų loginės grandinės

A	B	C	D	X
0	0	0	0	0
0	1	0	0	0
		...		
1	1	1	0	0
1	1	1	1	1





Grandinė bet kokiai loginei funkcijai apskaičiuoti

x_1	x_2	...	x_n	$f(x_1, x_2, \ldots, x_n)$	**konjunkcija**
0	0	...	0	1	$\overline{x_1} \cdot \overline{x_2} \cdot \ldots \cdot \overline{x_n}$
0	1	...	0	1	$\overline{x_1} \cdot x_2 \cdot \ldots \cdot \overline{x_n}$
0	1	...	0	0	0
0	1	...	0	0	0
		...			...
1	1	...	0	1	$x_1 \cdot x_2 \cdot \ldots \cdot \overline{x_n}$
1	1	...	1	0	0

Disjunkcinė normalioji forma (DNF):

$$\begin{aligned} f(x_1, x_2, \ldots, x_n) = \quad & \overline{x_1} \cdot \overline{x_2} \cdot \ldots \cdot \overline{x_n} \\ \vee \quad & \overline{x_1} \cdot x_2 \cdot \ldots \cdot \overline{x_n} \\ \vee \quad & \ldots \\ \vee \quad & x_1 \cdot x_2 \cdot \ldots \cdot \overline{x_n} \end{aligned}$$

Pavyzdys: funkcijos XOR (griežtosios disjunkcijos) sintezė

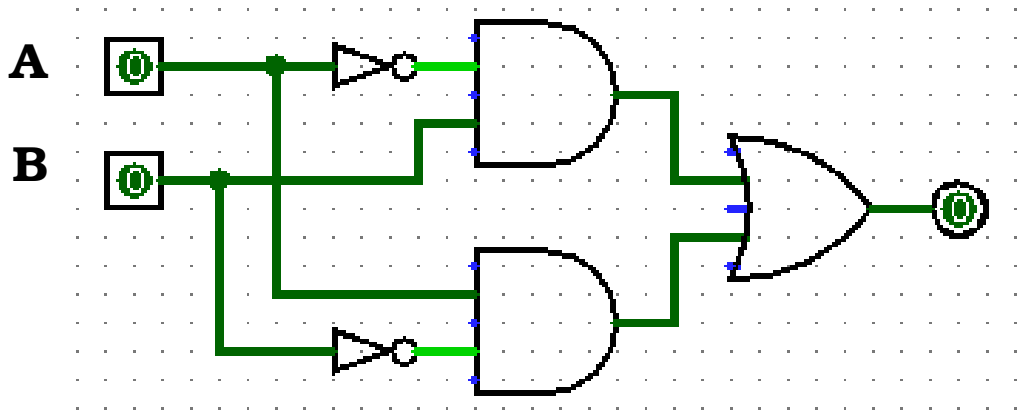
A	**B**	**X**	**konjunkcija**
0	0	0	
0	1	1	$\overline{A} \cdot B$
1	0	1	$A \cdot \overline{B}$
1	1	0	

DNF: $X = \overline{A}B \vee A\overline{B}$

Pavyzdys: funkcijos XOR (griežtosios disjunkcijos) sintezė

A	B	X	konjunkcija
0	0	0	
0	1	1	$\overline{A} \cdot B$
1	0	1	$A \cdot \overline{B}$
1	1	0	

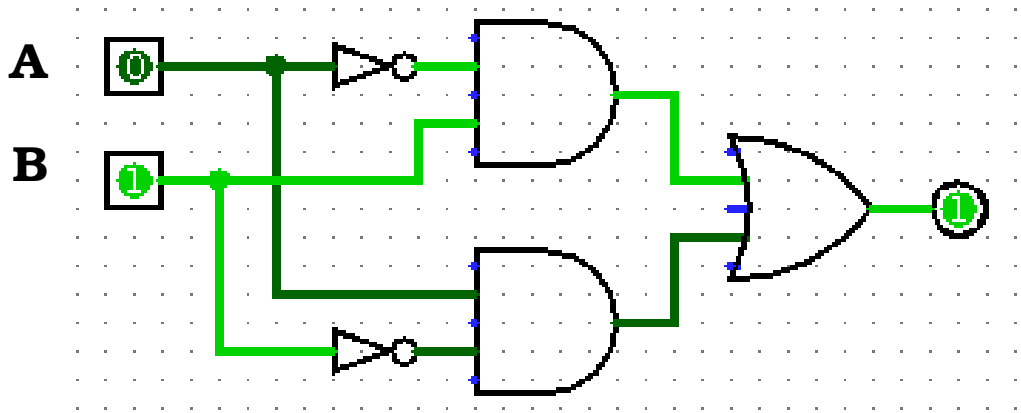
DNF: $X = \overline{A}B \vee A\overline{B}$



Pavyzdys: funkcijos XOR (griežtosios disjunkcijos) sintezė

A	B	X	konjunkcija
0	0	0	
0	1	1	$\overline{A} \cdot B$
1	0	1	$A \cdot \overline{B}$
1	1	0	

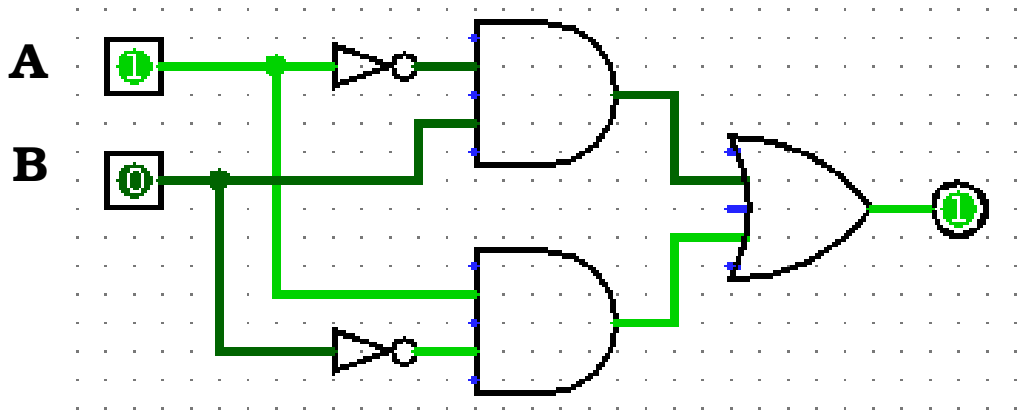
DNF: $X = \overline{A}B \vee A\overline{B}$



Pavyzdys: funkcijos XOR (griežtosios disjunkcijos) sintezė

A	B	X	konjunkcija
0	0	0	
0	1	1	$\overline{A} \cdot B$
1	0	1	$A \cdot \overline{B}$
1	1	0	

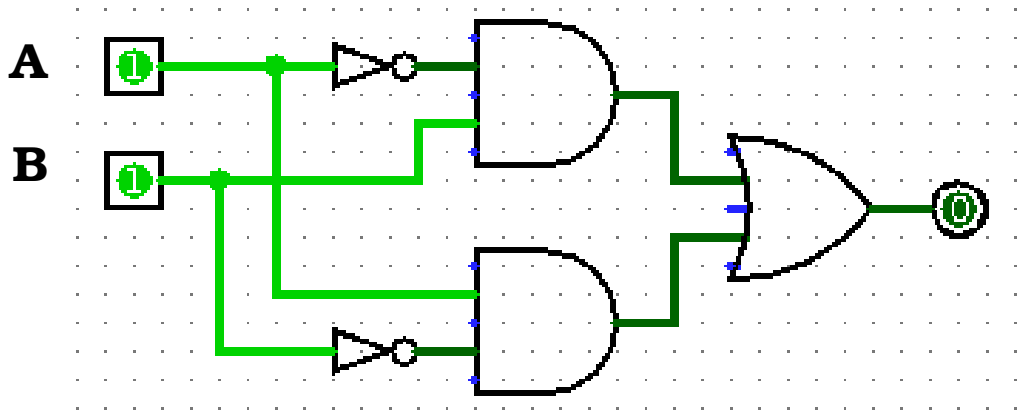
DNF: $X = \overline{A}B \vee A\overline{B}$



Pavyzdys: funkcijos XOR (griežtosios disjunkcijos) sintezė

A	B	X	konjunkcija
0	0	0	
0	1	1	$\overline{A} \cdot B$
1	0	1	$A \cdot \overline{B}$
1	1	0	

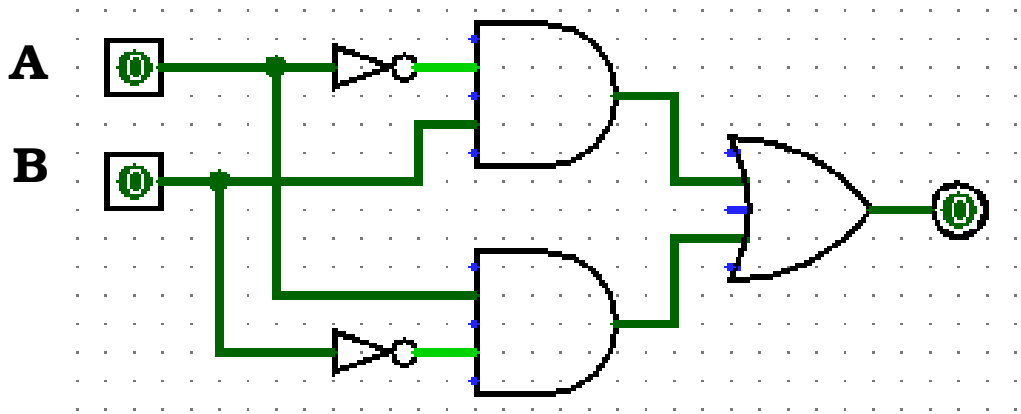
DNF: $X = \overline{A}B \vee A\overline{B}$

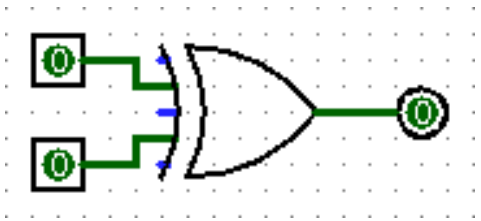


Pavyzdys: funkcijos XOR (griežtosios disjunkcijos) sintezė

A	B	X	konjunkcija
0	0	0	
0	1	1	$\overline{A} \cdot B$
1	0	1	$A \cdot \overline{B}$
1	1	0	

DNF: $X = \overline{A}B \vee A\overline{B}$

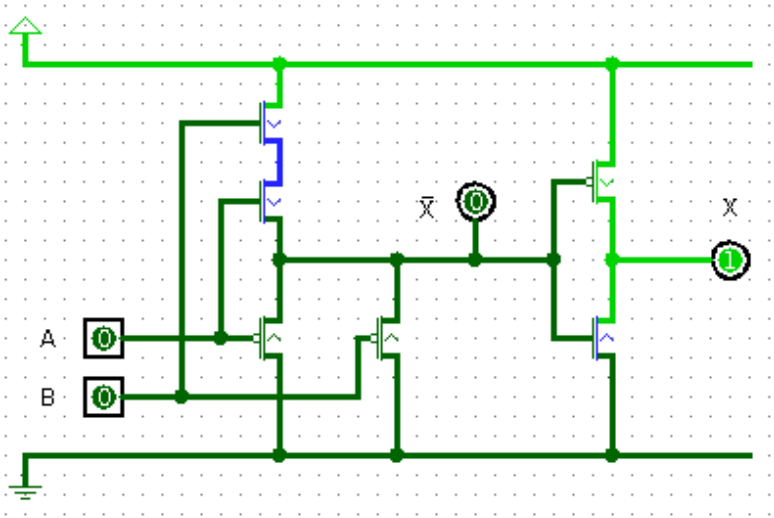


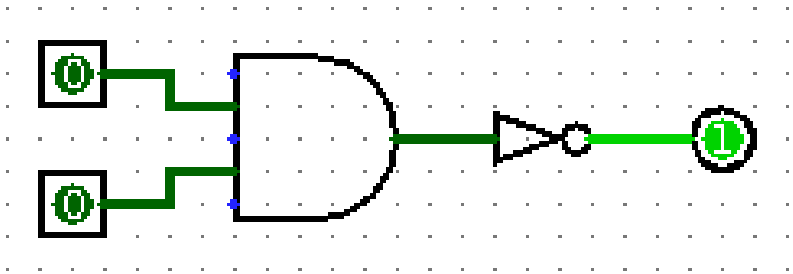


Loginis ventilis IR-NE

$$X = f(A, B) = \overline{AB}$$

A	B	X
0	0	1
0	1	1
1	0	1
1	1	0

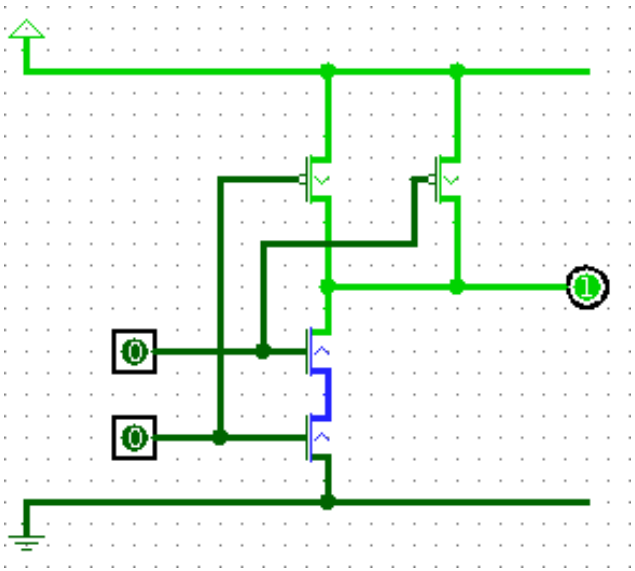


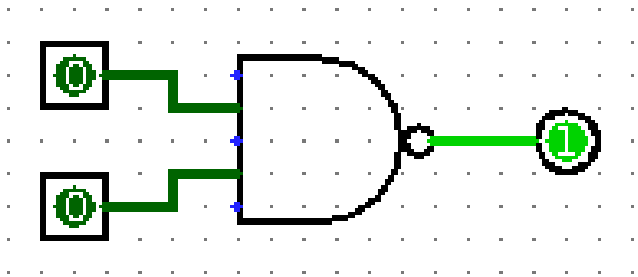


Loginis ventilis IR-NE

$$X = f(A, B) = \overline{AB}$$

A	B	X
0	**0**	**1**
0	1	1
1	0	1
1	1	0

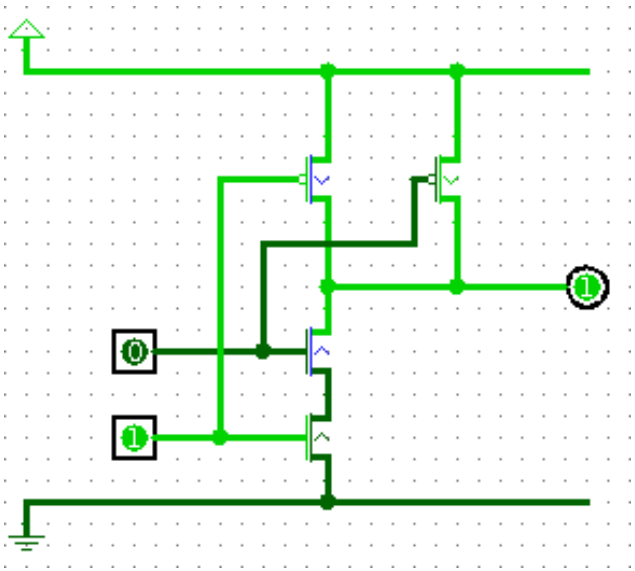


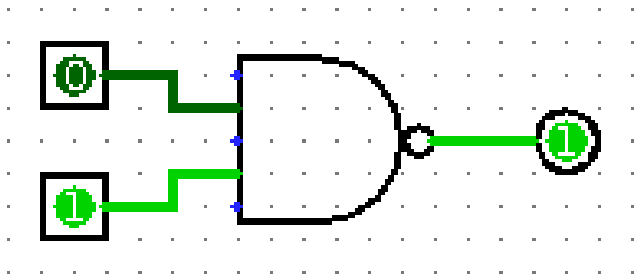


Loginis ventilis IR-NE

$$X = f(A, B) = \overline{AB}$$

A	B	X
0	0	1
0	**1**	**1**
1	0	1
1	1	0

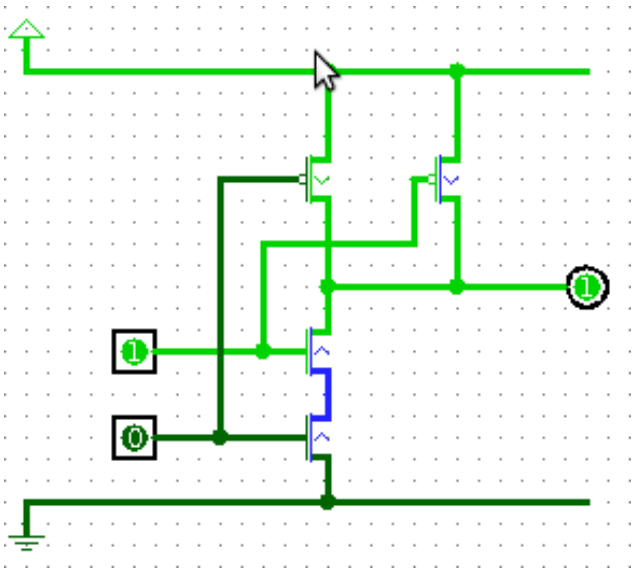


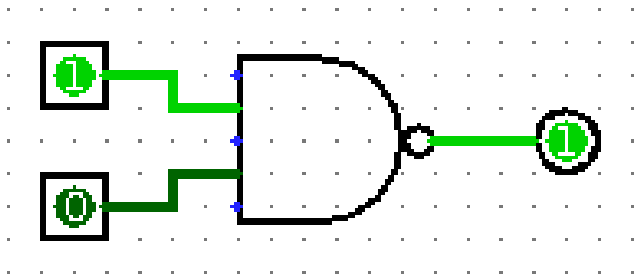


Loginis ventilis IR-NE

$$X = f(A, B) = \overline{AB}$$

A	B	X
0	0	1
0	1	1
1	**0**	**1**
1	1	0

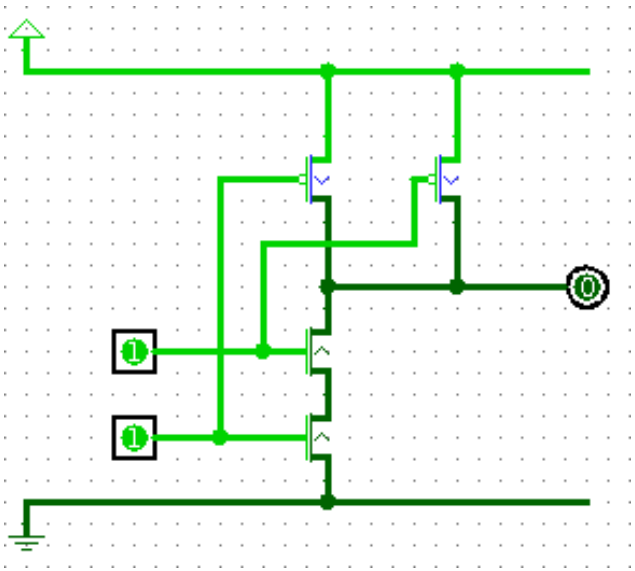


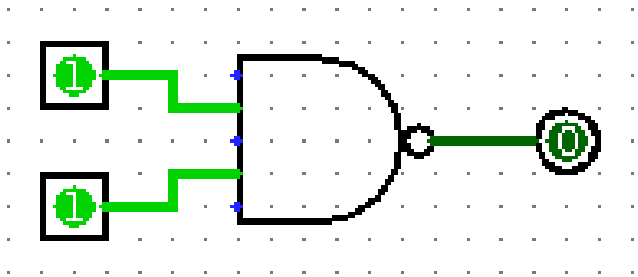


Loginis ventilis IR-NE

$$X = f(A, B) = \overline{AB}$$

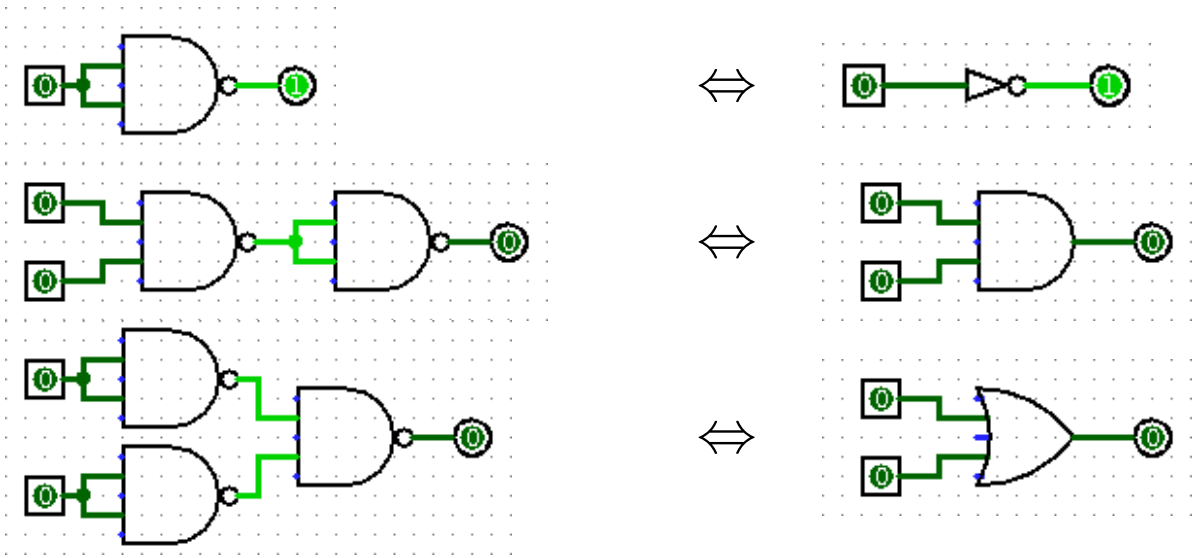
A	B	X
0	0	1
0	1	1
1	0	1
1	**1**	**0**





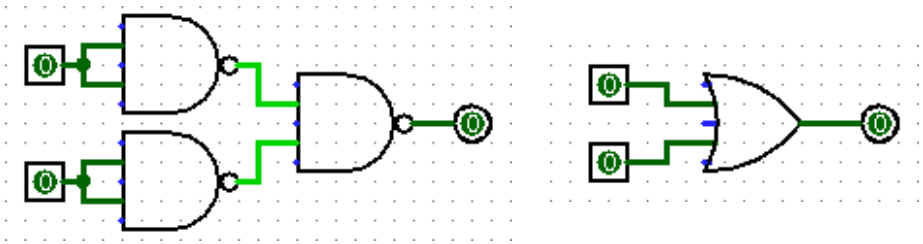
NAND – universalus loginis ventilis

A	B	X
0	0	1
0	1	1
1	0	1
1	1	0



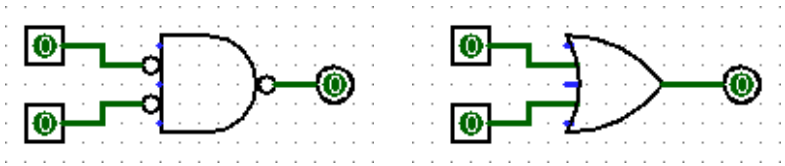
Dualūs ventiliai

A	B	$\overline{A}$	$\overline{B}$	$\overline{A \cdot B}$	$\overline{A} \cdot \overline{B}$	$\overline{\overline{A} \cdot \overline{B}}$	$A \vee B$
0	0	1	1	1	1	0	0
0	1	1	0	1	0	1	1
1	0	0	1	1	0	1	1
1	1	0	0	0	0	1	1



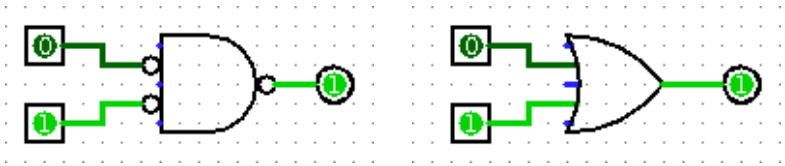
Dualūs ventiliai

A	B	$\overline{A}$	$\overline{B}$	$\overline{A \cdot B}$	$\overline{A} \cdot \overline{B}$	$\overline{\overline{A} \cdot \overline{B}}$	$A \vee B$
0	0	1	1	1	1	0	0
0	1	1	0	1	0	1	1
1	0	0	1	1	0	1	1
1	1	0	0	0	0	1	1



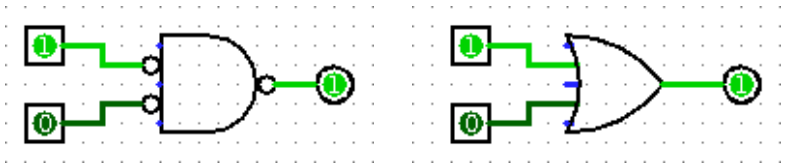
Dualūs ventiliai

A	B	$\overline{A}$	$\overline{B}$	$\overline{A \cdot B}$	$\overline{A} \cdot \overline{B}$	$\overline{\overline{A} \cdot \overline{B}}$	$A \vee B$
0	0	1	1	1	1	0	0
0	1	1	0	1	0	1	1
1	0	0	1	1	0	1	1
1	1	0	0	0	0	1	1



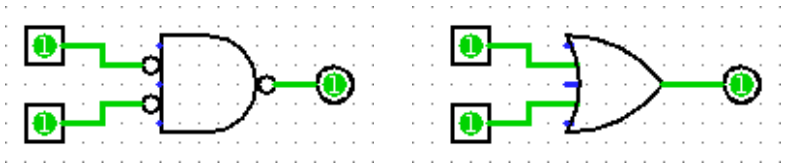
Dualūs ventiliai

A	B	$\overline{A}$	$\overline{B}$	$\overline{A \cdot B}$	$\overline{A} \cdot \overline{B}$	$\overline{\overline{A} \cdot \overline{B}}$	$A \vee B$
0	0	1	1	1	1	0	0
0	1	1	0	1	0	1	1
1	0	0	1	1	0	1	1
1	1	0	0	0	0	1	1



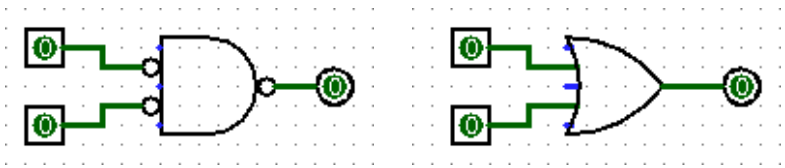
Dualūs ventiliai

A	B	$\overline{A}$	$\overline{B}$	$\overline{A \cdot B}$	$\overline{A} \cdot \overline{B}$	$\overline{\overline{A} \cdot \overline{B}}$	$A \vee B$
0	0	1	1	1	1	0	0
0	1	1	0	1	0	1	1
1	0	0	1	1	0	1	1
1	1	0	0	0	0	1	1



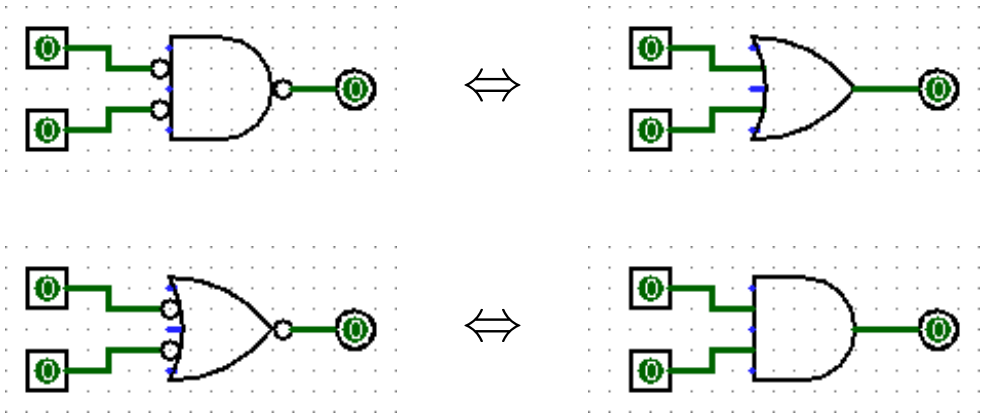
Dualūs ventiliai

A	B	$\overline{A}$	$\overline{B}$	$\overline{A \cdot B}$	$\overline{A} \cdot \overline{B}$	$\overline{\overline{A} \cdot \overline{B}}$	$A \vee B$
0	0	1	1	1	1	0	0
0	1	1	0	1	0	1	1
1	0	0	1	1	0	1	1
1	1	0	0	0	0	1	1



$$
\begin{aligned}
A \vee B &= \overline{\overline{A} \cdot \overline{B}} \\
\overline{\overline{A}} &= A \\
\overline{A \vee B} &= \overline{\overline{A} \cdot \overline{B}} = \overline{A} \cdot \overline{B}
\end{aligned}
$$

Dualūs loginiai elementai, De Morgano dėsniai

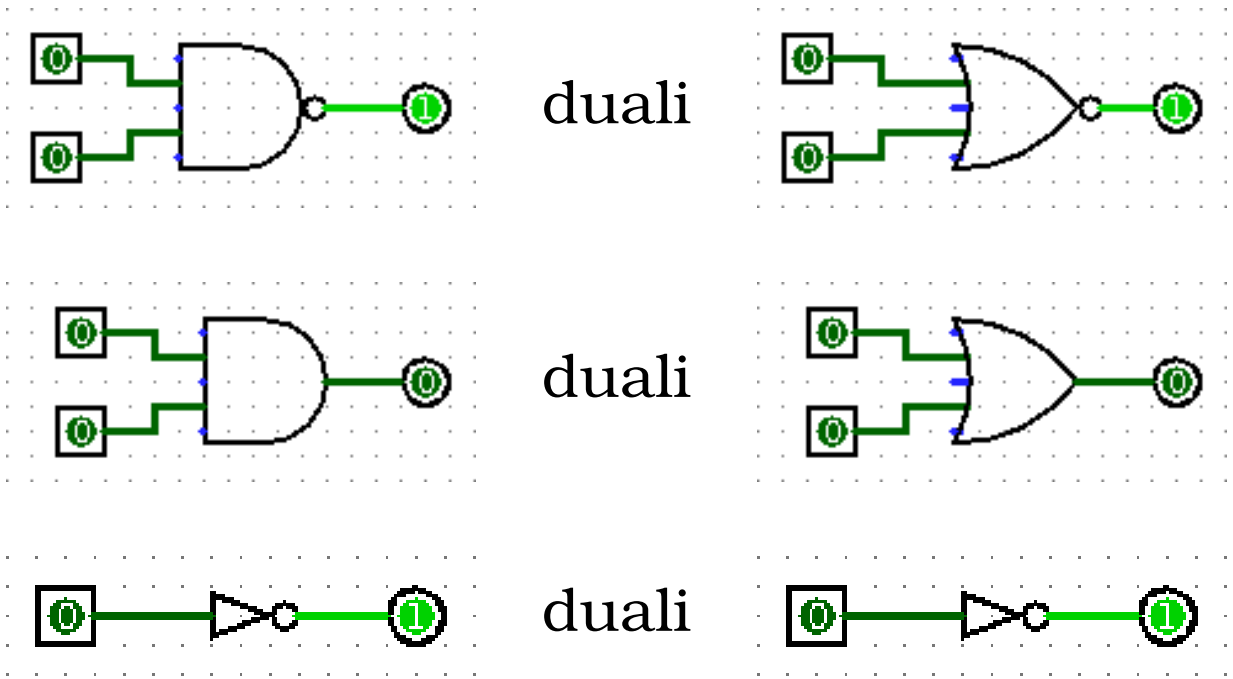


$$\left.\begin{array}{rcl} \overline{\overline{A}\cdot\overline{B}} & = & A\vee B \\ \overline{\overline{A}\vee\overline{B}} & = & A\cdot B \end{array}\right\} \text{ dualios funkcijos}$$

$$\left.\begin{array}{rcl} \overline{A\vee B} & = & \overline{A}\cdot\overline{B} \\ \overline{A\cdot B} & = & \overline{A}\vee\overline{B} \end{array}\right\} \text{ De Morgano dėsniai}$$

$$h \text{ yra duali } f \underset{\text{def}}{\Leftrightarrow} h(x_1,x_2,\ldots,x_n)=\overline{f(\overline{x_1},\overline{x_2},\ldots,\overline{x_n})}$$

Dar vienas universalus elementas: NOR



Pilna loginių funkcijų sistemos

Superpozicija:

$$h(x_1, \ldots, x_n) = f(g_1(x_1, \ldots, x_n), \ldots, g_m(x_1, \ldots, x_n))$$

Pavyzdys:

$$\mathrm{NOT}(\mathrm{AND}(x_1, \mathrm{NOT}(x_2)) \underset{\text{def}}{\equiv} \overline{x_1 \overline{x_2}} = \overline{x_1} \vee \overline{\overline{x_2}} = \overline{x_1} \vee x_2$$

Pilnos funkcijų sistemos (pavyzdžiai):

$$\{\mathrm{AND}, \mathrm{OR}, \mathrm{NOT}\} = \{\cdot, \vee, \overline{X}\} \quad \text{(neminimali!)}$$
$$\{\mathrm{AND}, \mathrm{NOT}\} = \{\cdot, \overline{X}\}$$
$$\{\mathrm{OR}, \mathrm{NOT}\} = \{\vee, \overline{X}\}$$
$$\{\mathrm{AND}, \mathrm{XOR}, 1\} = \{\cdot, \oplus, 1\}$$
$$\{\mathrm{OR}, \mathrm{XOR}, 1\} = \{\vee, \oplus, 1\}$$
$$\{\mathrm{NAND}\} = \{\uparrow\} \quad \text{Sheffer stroke}$$
$$\{\mathrm{NOR}\} = \{\downarrow\} \quad \text{Peirce's arrow/Quine's dagger}$$

Uždaros Būlio funkcijų klasės

Funkcijų klasės, uždaros superpozicijos atžvilgiu:

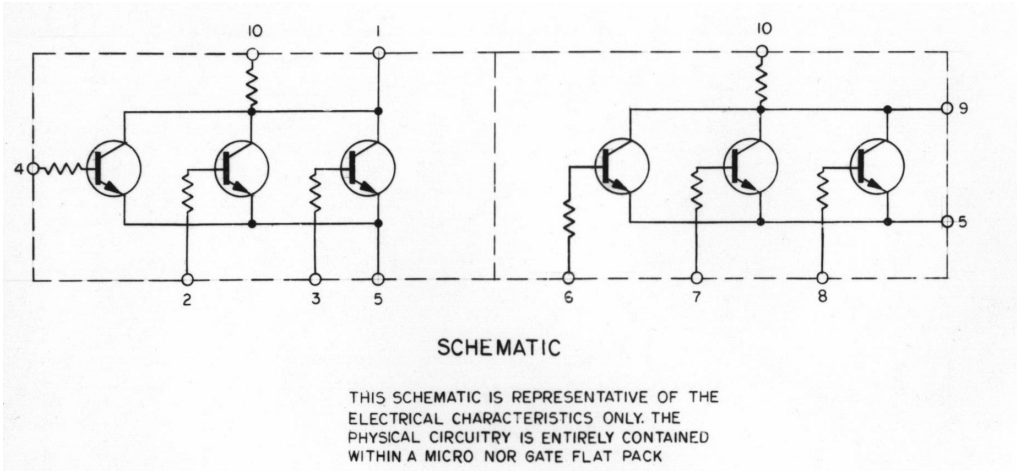
1. T_0: išsaugančios nulį:
$$f(0,0,\ldots,0) = 0$$
2. T_1: išsaugančios vienetą:
$$f(1,1,\ldots,1) = 1$$
3. S: dualios sau:
$$f(x_1,x_2,\ldots,x_n) = \overline{f(\overline{x_1},\overline{x_2},\ldots,\overline{x_n})}$$
4. L: tiesinės:
$$f(x_1,x_2,\ldots,x_n) = a_0 \oplus a_1x_1 \oplus \cdots \oplus a_nx_n$$
5. M: monotoninės:
$$\vec{a} \underset{\text{def}}{\equiv} (a_1,a_2,\ldots,a_n)$$
$$\vec{a} \leq \vec{b} \underset{\text{def}}{\Leftrightarrow} \forall i \in \{1,\ldots,n\} : a_i \leq b_i$$
$$f \in M \underset{\text{def}}{\Leftrightarrow} \vec{a} \leq \vec{b} \Rightarrow f(\vec{a}) \leq f(\vec{b})$$

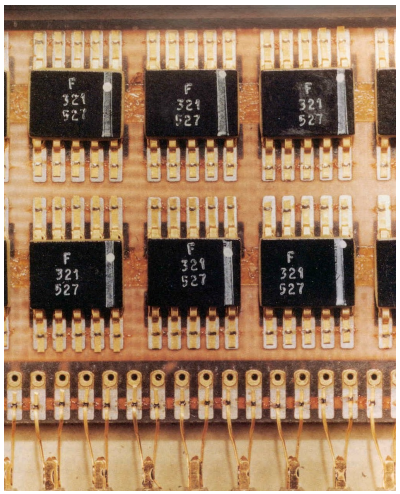
Posto teorema

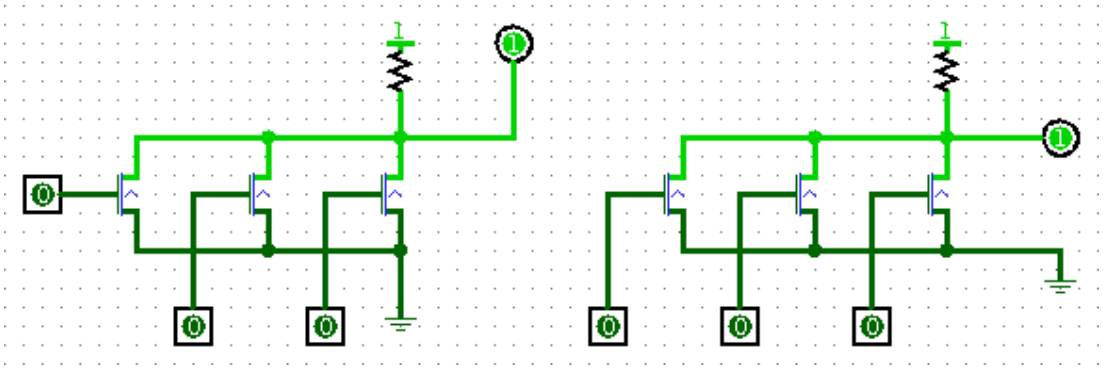
Būlio funkcijų aibė $B = \{f_1, f_2, \ldots, f_n\}$ yra pilna superpozicijos atžvilgiu **tada ir tik tada**, jeigu:

1. joje yra funkcija, neišsauganti nulio:
$$\exists f_i \in B : f_i \notin T_0$$
2. joje yra funkcija, neišsauganti vieneto:
$$\exists f_i \in B : f_i \notin T_1$$
3. joje yra funkcija, neduali sau pačiai:
$$\exists f_i \in B : f_i \notin S$$
4. joje yra netiesinė funkcija:
$$\exists f_i \in B : f_i \notin L$$
5. joje yra nemonotoninė funkcija:
$$\exists f_i \in B : f_i \notin M$$

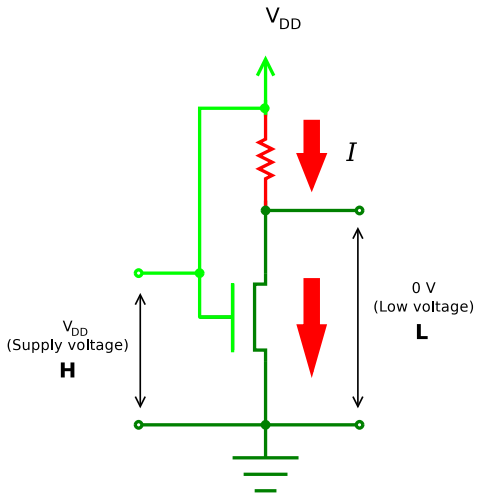
“Apolono” valdymo kompiuterio ventiliai

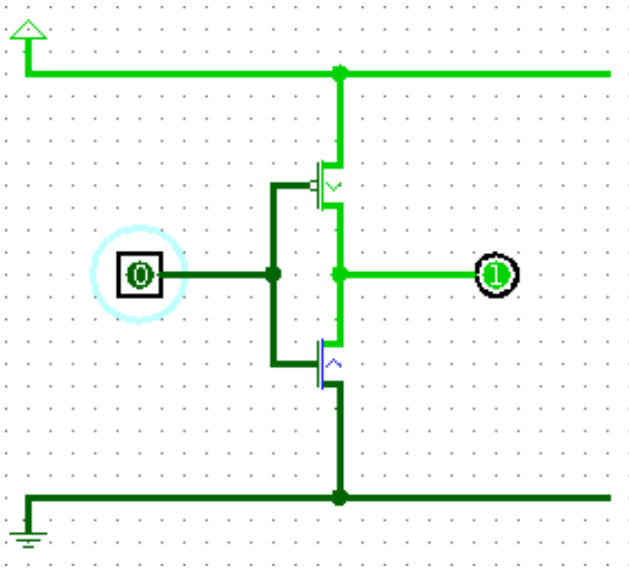


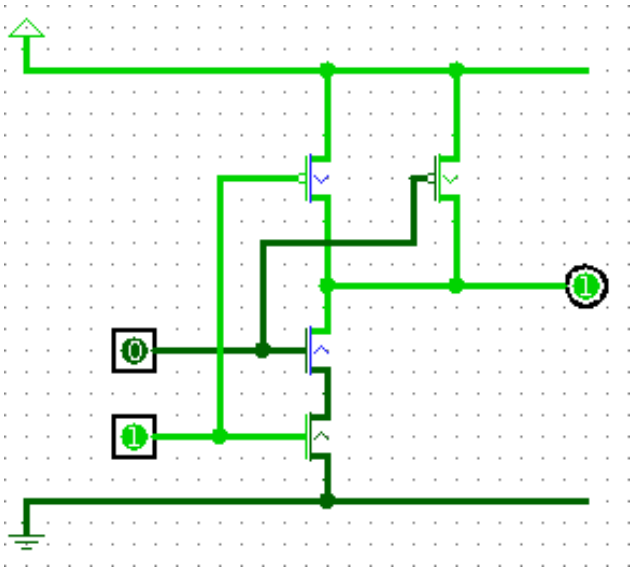


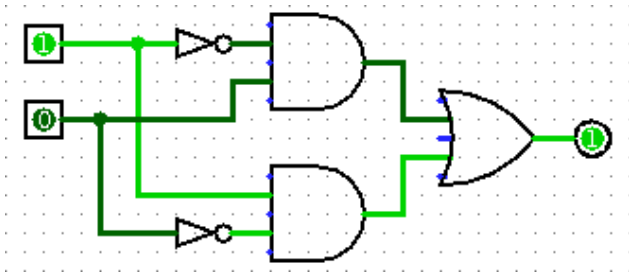


Ką mes iki šiol pasiekėme ...









Apibendrinimas

- Skaitmenines grandines galima aprašyti Būlio algebros pagalba;
- Bet kokia Būlio funkcija gali būti sukonstruota iš pilnos funkcijų aibės (žr. Posto teoremą); loginės funkcijos NAND ir NOR kiekviena pati sudaro pilną funkcijų aibę ir leidžia sukonstruoti bet kokią loginę grandinę;
- Visas logines funkcijas gali realizuoti elektriniai perjungikliai (pvz. KMOP lauko tranzistoriai);
- Visus kompiuterio mazgus galima pagaminti iš valdomų perjungiklių; šiuolaikiniuose kompiuteriuose kaip perjungikliai naudojami KMOP lauko tranzistoriai.