$$\frac{\partial E}{\partial u_{ij}} = U \cdot S - R$$

$$\frac{\partial F}{\partial u_{ij}} = U \cdot L$$

$$\frac{\partial (E+F)}{\partial u_{ij}} = US - R + U \cdot L = 0$$

$$U\,(S+L) = R$$

$$\frac{\partial^2 G}{\partial u_{mk}\partial u_{ij}} = ?$$

$$\frac{\partial G}{\partial u_{ij}} = \sum_k u_{ik} \overbrace{\sum_n x_{kn}x_{jn}}^{S_{kj}} - \overbrace{\sum_n y_{in}x_{jn}}^{R} + \sum_k u_{ik}\, l_{kj} =$$

$$= \sum_k u_{ik}\left(\sum_n x_{kn}x_{jn} + l_{kj}\right) - \sum_n y_{in}x_{jn}$$

$$\frac{\partial}{\partial u_{ml}}\left(\frac{\partial G}{\partial u_{ij}}\right) = \frac{\partial}{\partial u_{ml}}\left(\sum_k u_{ik}\left(\sum_n x_{kn}x_{jn} + l_{kj}\right) - \overset{\frac{\partial}{\partial u_{ij}} = 0}{\cancel{\sum_n y_{in}x_{jn}}}\right)$$

$$= \sum_k \frac{\partial u_{ik}}{\partial u_{ml}}\left(\sum_n x_{kn}x_{jn} + l_{kj}\right) =$$

$$= \sum_k \delta_{im}\delta_{kl}\left(\sum_n x_{\underset{l}{k}n}x_{jn} + l_{\underset{l}{k}j}\right) =$$

$$= \delta_{im}\left(\underbrace{\sum_n x_{ln}x_{jn}}_{S_{lj}} + \underbrace{\lambda_{lj}}_{L_{lj}}\right)$$

$$S + L$$

Nuosavi vektoriai ir reikšmės

$$A \cdot \vec{x} = \lambda \vec{x}, \quad A = [a_{ij}], \quad \vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = [x_j]$$

$$\vec{x} \neq \vec{0} \qquad \lambda \in \mathbb{R}$$

$$(A - \lambda E)\vec{x} = \vec{0}$$

$$\Downarrow$$

$$\det(A - \lambda E) = 0$$

$$\det \begin{bmatrix} a_{11}-\lambda & a_{12} & a_{13} \\ a_{21} & a_{22}-\lambda & a_{23} \\ a_{31} & a_{32} & a_{33}-\lambda \end{bmatrix} = 0$$

$$p(\lambda^3, \lambda^2, \lambda, 1) = 0$$

$$A^T = A \qquad A^+ = \overline{A^T} = A$$

(Hermitian matrix)

Ermito matrica

2010.05.11

Simetrinių matricų t.v. ir t.r.

$A^{+} = \overline{A^{T}}$

$$(\vec{a}, \vec{b}) = [a_i]^{+}[b_i] = [\bar{a}_1, \bar{a}_2, \dots, \bar{a}_n] \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

$$= \vec{a}^{+} \cdot \vec{b}$$

$$(\vec{a}, A\vec{b}) = \vec{a}^{+}(A\vec{b}) = (\vec{a}^{+}A)\vec{b} =$$

$$= (A^{+}\vec{a})^{+}\vec{b} = (A^{+}\vec{a}, \vec{b})$$

$$(AB)^{T} = B^{T}A^{T}, \quad (AB)^{+} = \overline{(AB)^{T}} = \overline{(B^{T}A^{T})} =$$

$$= (\overline{B^{T}} \cdot \overline{A^{T}}) = B^{+}A^{+}$$

$A^{+} = A$

$$(\vec{a}, A\vec{b}) = (A^{+}\vec{a}, \vec{b}) = (A\vec{a}, \vec{b})$$

T tegul $\vec{x}$ yra s.m. A t.v.: $\vec{x} \neq \vec{0}$

$$A\vec{x} = \lambda\vec{x}$$

$$(\vec{x}, \vec{x}) = |\vec{x}| \in \mathbb{R}$$

$\boxed{\text{Visos s.m. t.v. yra } \underline{\text{realios}}}$

$$(\cancel{A}\vec{x}, A\vec{x}) = (\vec{x}, \lambda\vec{x}) = \lambda|\vec{x}|$$

$$\|$$

$$(A\vec{x}, \vec{x}) = (\lambda\vec{x}, \vec{x}) = [\lambda x_i]^{T} \cdot [x_i] = \bar{\lambda}(\vec{x}, \vec{x}) =$$

$$= \bar{\lambda}|\vec{x}|$$

2010.05.11

Simetrinių matricų

(n.) t.v. ir t. (n.) r.

$$\overline{\lambda}\,|\vec{x}| = \lambda\,|\vec{x}|$$

$$|\vec{x}|\in\mathbb{R},\ |\vec{x}|\neq 0 \Rightarrow \lambda\in\mathbb{R}$$

II

S.m. t.(n.)v. yra ortogonalūs

$$\vec{x},\vec{y}\neq\vec{0}, \quad A\vec{x}=\lambda\vec{x}, \quad A\vec{y}=\mu\vec{y}$$

$$\lambda\neq\mu$$

$$(\vec{x},\vec{y})=?$$

$$\lambda(\vec{x},\vec{y}) = (\lambda\vec{x},\vec{y}) = (A\vec{x},\vec{y}) =$$

$$= (\vec{x},A^{+}\vec{y}) \underset{A=A^{+}}{=} (\vec{x},A\vec{y}) = (\vec{x},\mu\vec{y}) =$$

$$= \mu(\vec{x},\vec{y})$$

$$\left.\begin{matrix}\lambda(\vec{x},\vec{y})=\mu(\vec{x},\vec{y})\\ \lambda\neq\mu\end{matrix}\right\} \Rightarrow (\vec{x},\vec{y})=0,$$

t.y.

$$\vec{x}\perp\vec{y}$$

Teigiamos matricos

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$$f(\vec{x}) = \vec{x}^T A \vec{x} \qquad A^+ = A$$

A teigiama, jei

$$\forall \vec{x} \neq \vec{0} \quad f(\vec{x}) = \vec{x}^T A \vec{x} > 0$$

o kas, jei $\vec{x}$ yra n. (t.) v.?

$$f(\vec{x}) = \vec{x}^T A \vec{x} = \vec{x}^T \lambda \vec{x} = \lambda (\vec{x}^T \vec{x}) = \lambda (\vec{x}, \vec{x}) =$$
$$= \lambda |\vec{x}|^2$$

$$|\vec{x}|^2 > 0 \Rightarrow \lambda > 0$$